Self-Organization from the Perspective of a Physicist

Udo Erdmann

Berlin, 26/03/2007

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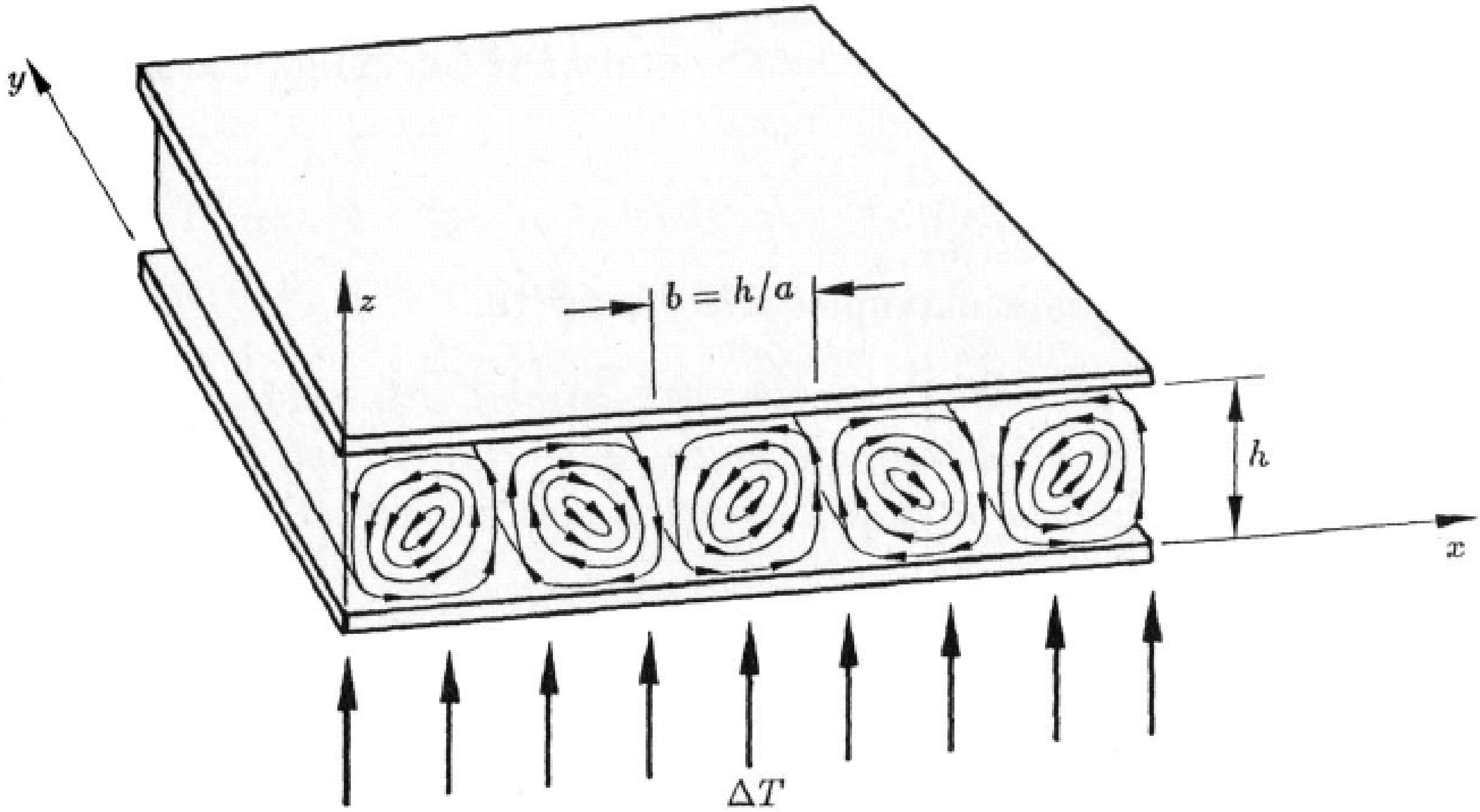
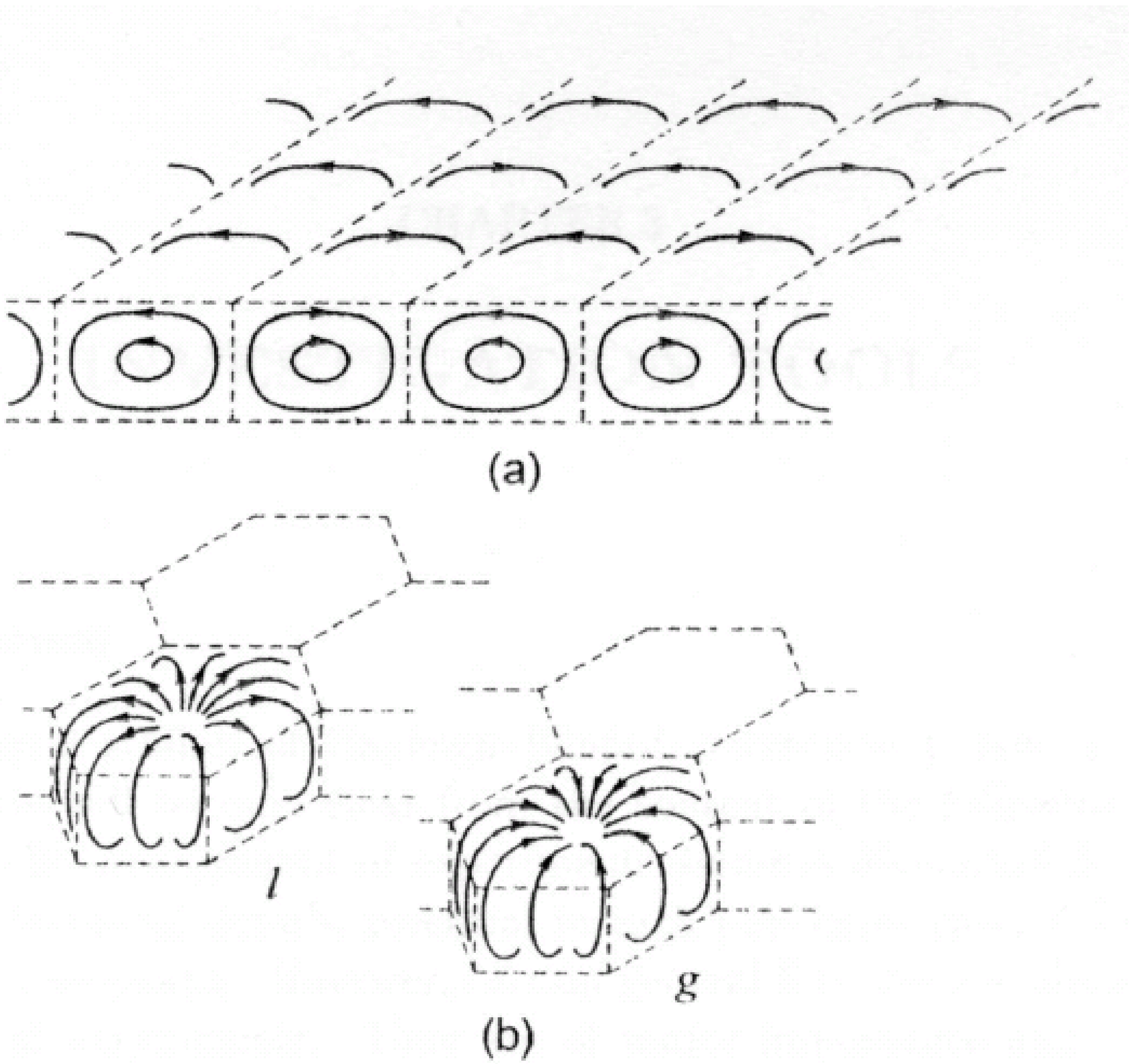
Rayleigh-Bénard Convection





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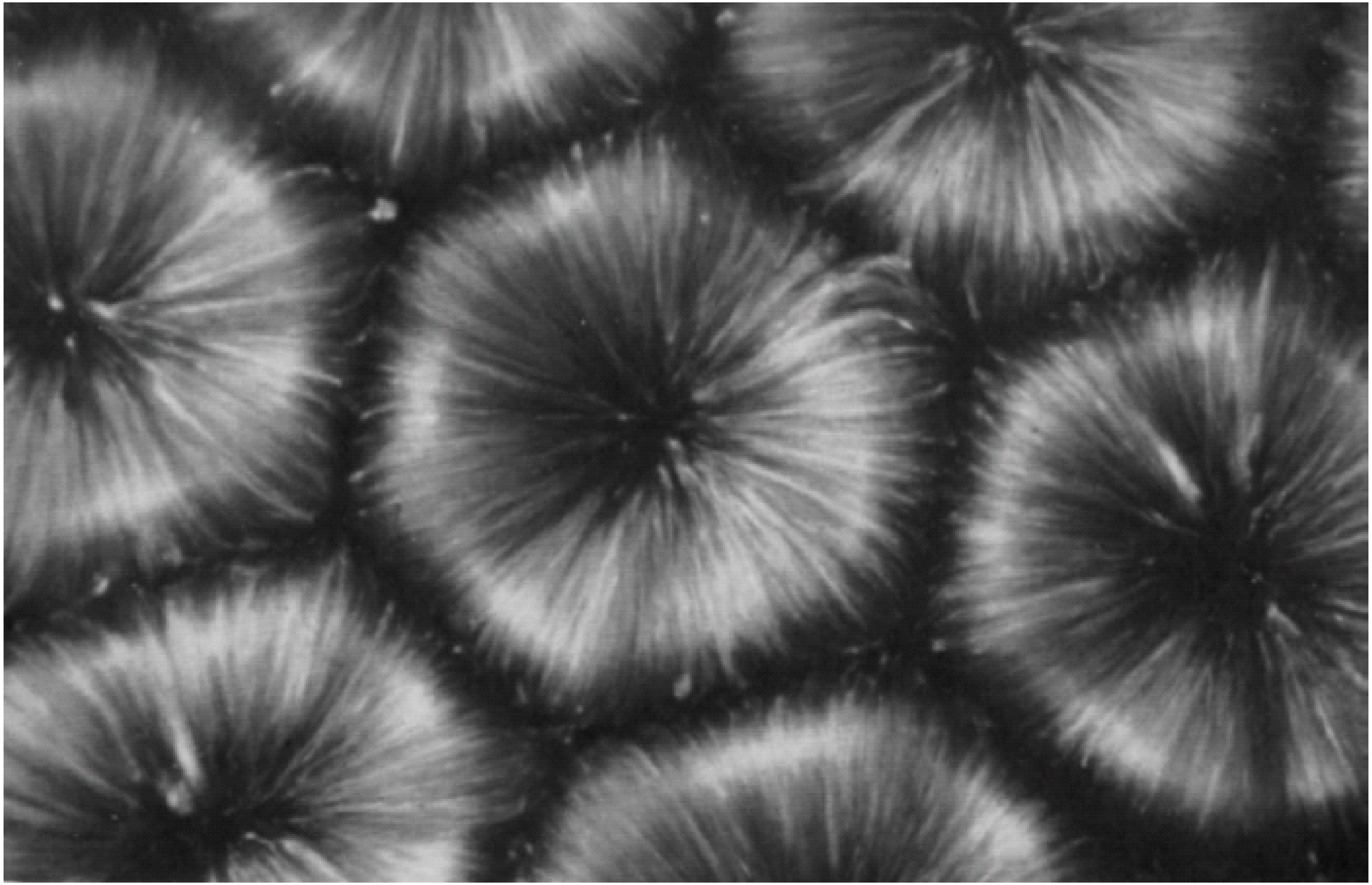
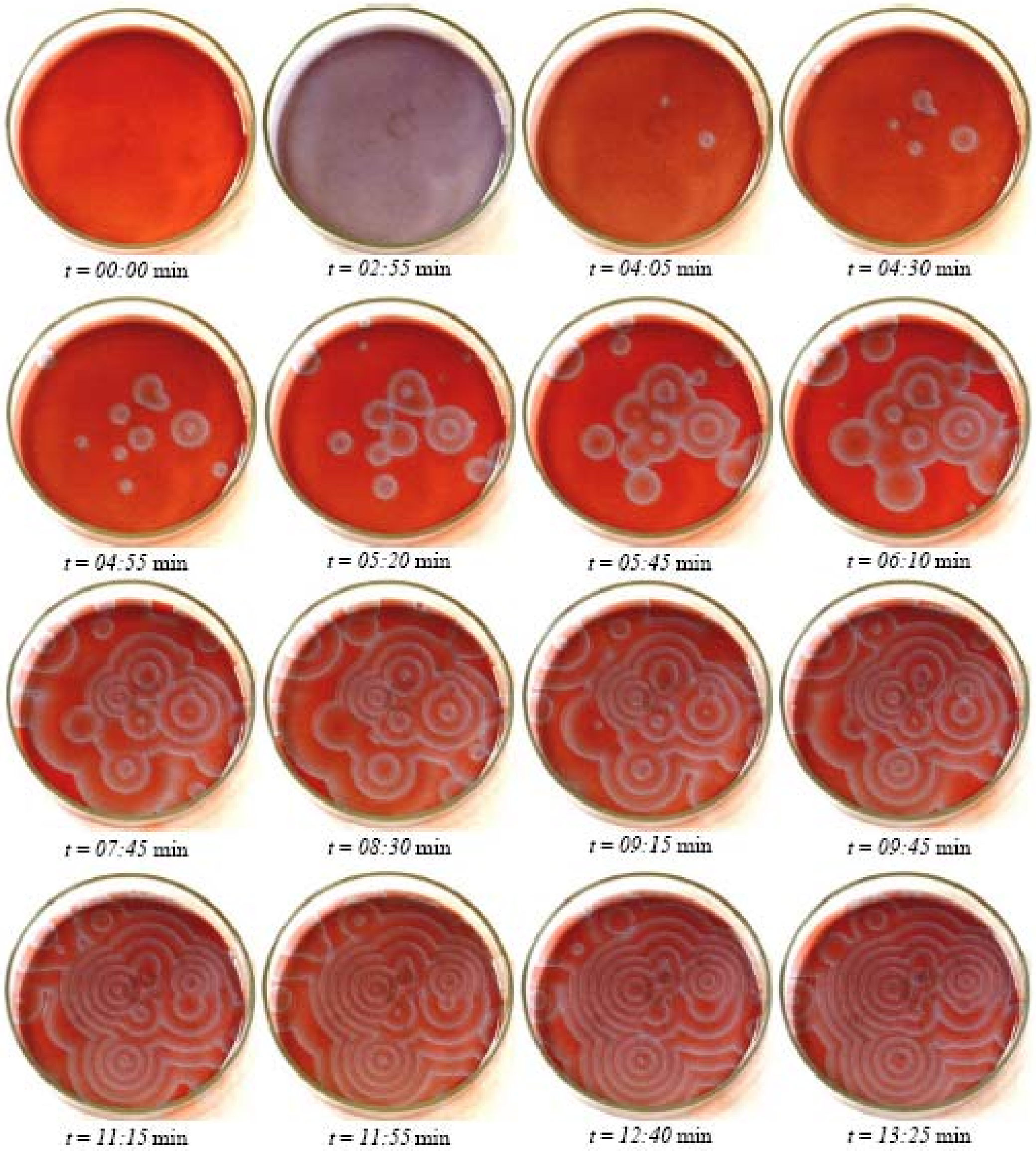
Bénard-Marangoni Effect



Belousov-Zhabotinsky-Reaktion

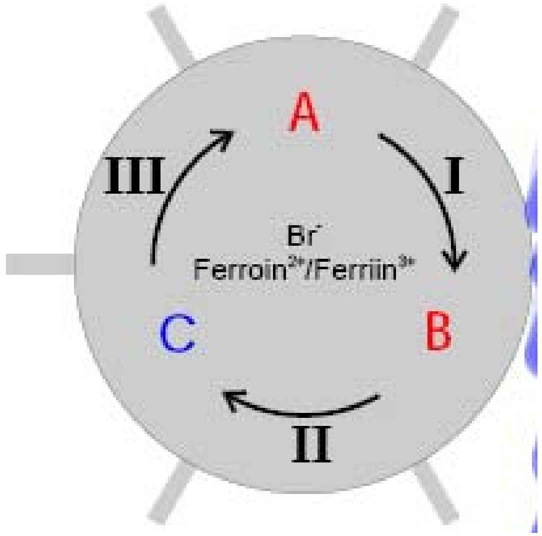


$$2Br^- + BrO_3^- + 3H^+ + 3HMal \longrightarrow 3HBrMal + H_2O$$

$$BrO_3^- + 4\,\text{Ferroin}^{2+} + HMal + 5H^+ \longrightarrow 4\,\text{Feriin}^{3+} + HBrMal + 3H_2O$$

$$4\,\text{Feriin}^{3+} + HBrMal + H_2O \longrightarrow 4\,\text{Ferroin}^{2+} + HCOOH + 2CO_2\uparrow + 5H^+ + Br^-$$

$$3BrO_3^- + 5HMal + 3H^+ \longrightarrow 3HBrMal + 2HCOOH + 4CO_2 + 5H_2O$$



light-sensitive reaction

Jan Krieger

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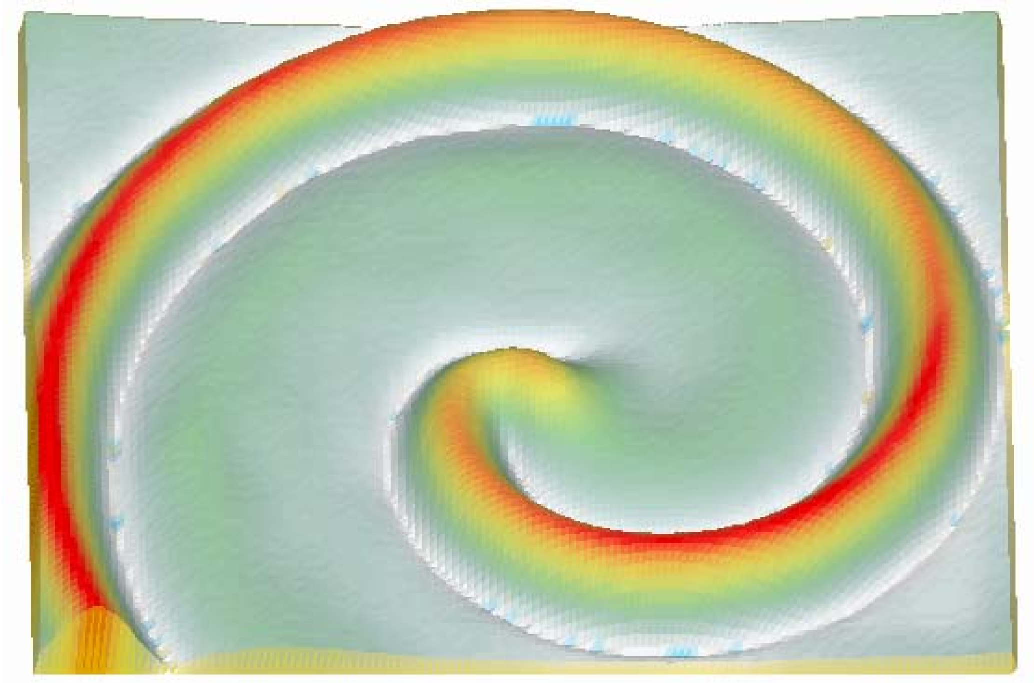
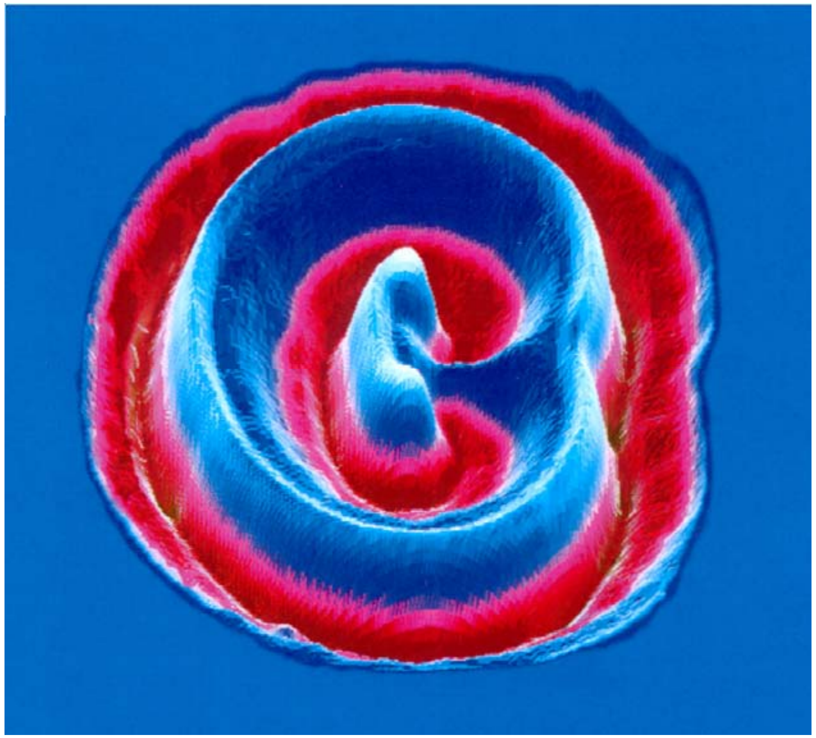
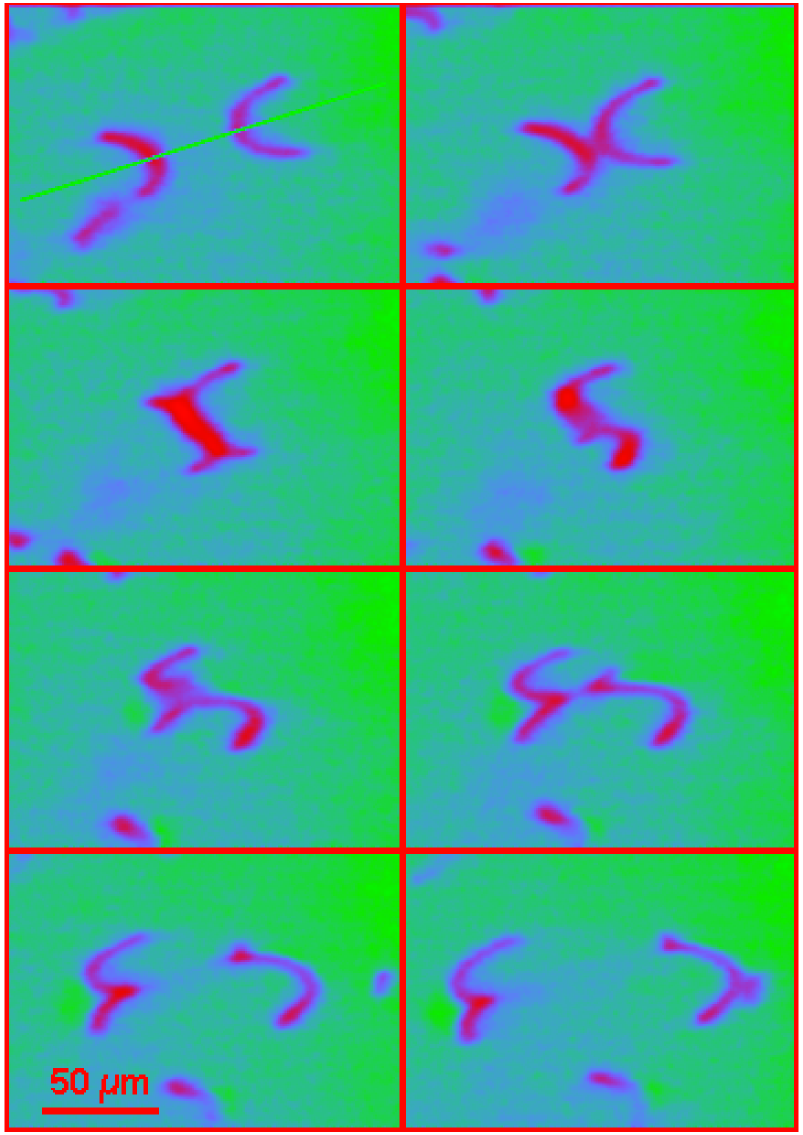
Belousov-Zhabotinsky-Reaktion



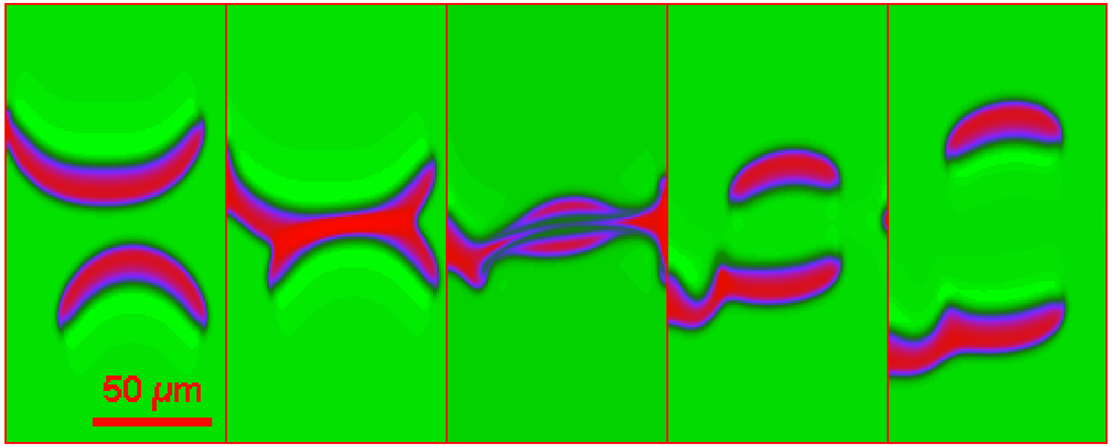


Group of „Dissipative Structures“,
H. Engel (TU Berlin)

CO on Platinum Surfaces



experiment



simulation

A. v. Oertzen, H.-H. Rotermund, A. S. Mikahilov, FHI Berlin

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Selforganization

- Open system
  Non-equilibrium
- Non-linearity and/or interacting species with different relaxation time scales
- Fluctuations

- Non-linear sytems offer more than one solution which can be obtained.
- Fluctuations allow the system to switch between the different possible solutions

Swarming, self-organized?

As we know from a lot of species, individuals tend to form groups.

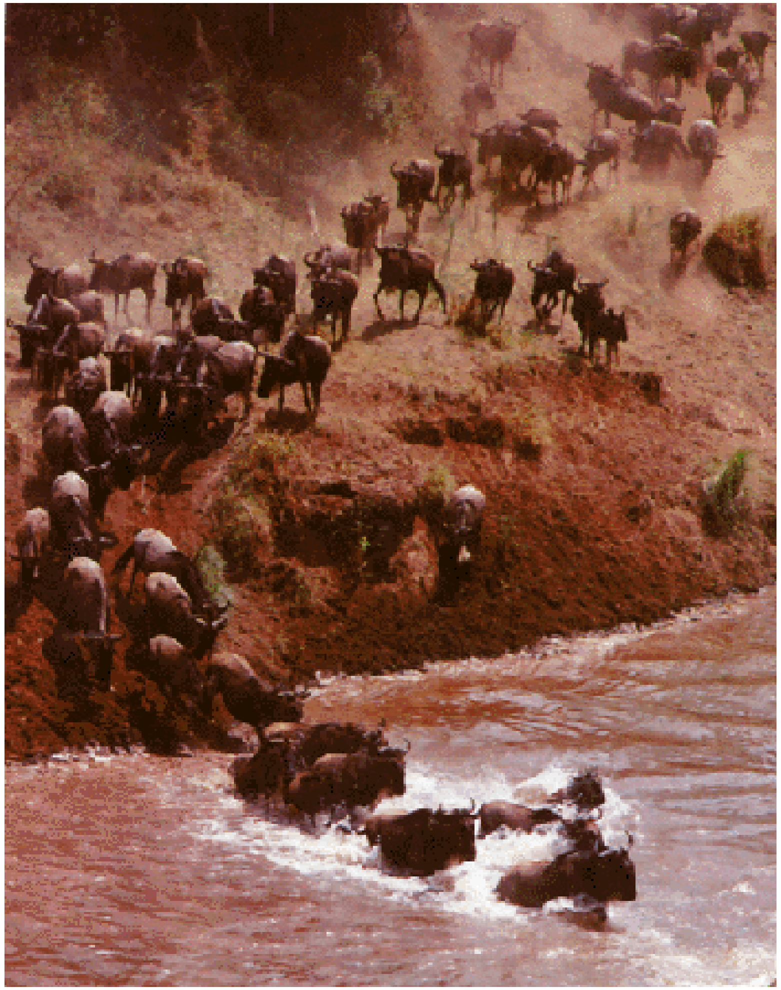
Within these groups coherent motion of the group itself can be observed.

- Wildebeest live in herds
- Fish form schools
- Birds fly in flocks
- Locusts move in large swarms
- ...

Wildebeests

Plate 3. Wildebeest massing in a grazing front on the Serengeti Plains. March 1973.

Buffalos



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Anchovis Trying to Survive

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Swarming?

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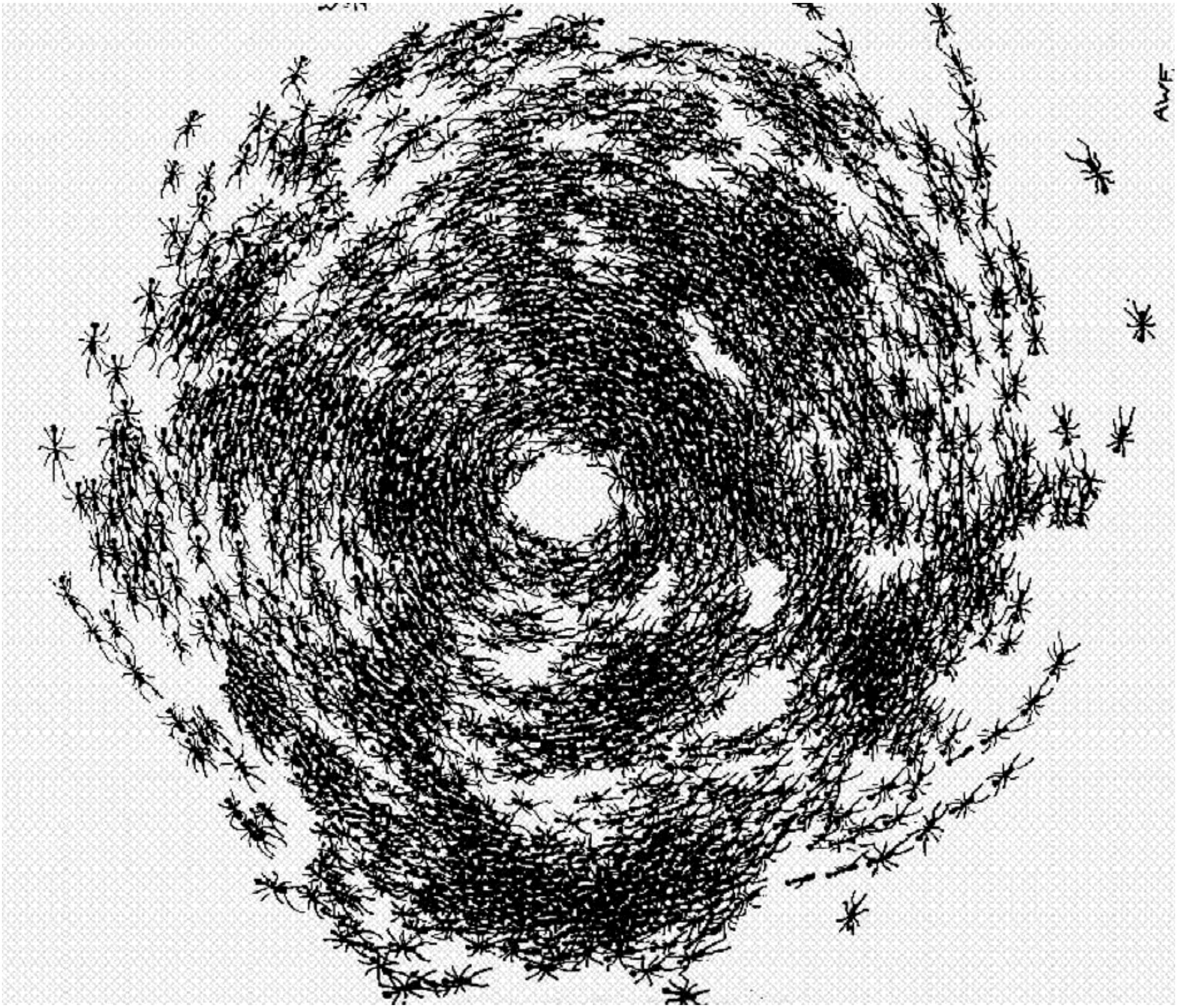
Flocks of Birds

Swarming of Army Ants



A swarm of Army ants runs in a circle for five days.
(T. C. Schneirla, 1948)

A. Aronson, E. Tobach, J. S. Rosenblatt & D. S. Lehrmann (Eds.): *Selected Writings of Theodore C. Schneirla*, Freeman & Co., San Francisco (1972)

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Collective Motion in Bacterial Colonies

PHYSICAL REVIEW E VOLUME 54, NUMBER 2 AUGUST 1996

Formation of complex bacterial colonies via self-generated vortices

András Czirók,[1] Eshel Ben-Jacob,[2] Inon Cohen,[2] and Tamás Vicsek[1,3]
[1]*Department of Atomic Physics, Eötvös University, Puskin u. 5-7, 1088 Budapest, Hungary*
[2]*School of Physics, Tel-Aviv University, 69978 Tel-Aviv, Israel*
[3]*Institute for Technical Physics, P.O. Box 76, 1325 Budapest, Hungary*

1995; revised manuscript received 29 March 1996)

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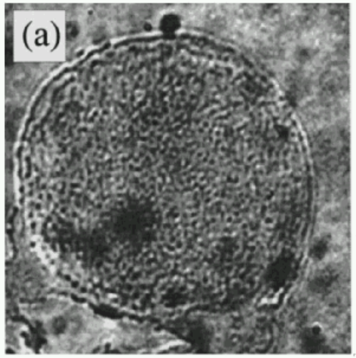


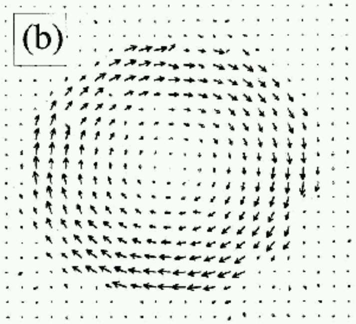


FIG. 2. Bright field micrograph of a single rotating droplet with a magnification of 500× (a) and the corresponding velocity field obtained by digitizing our video recordings (b).

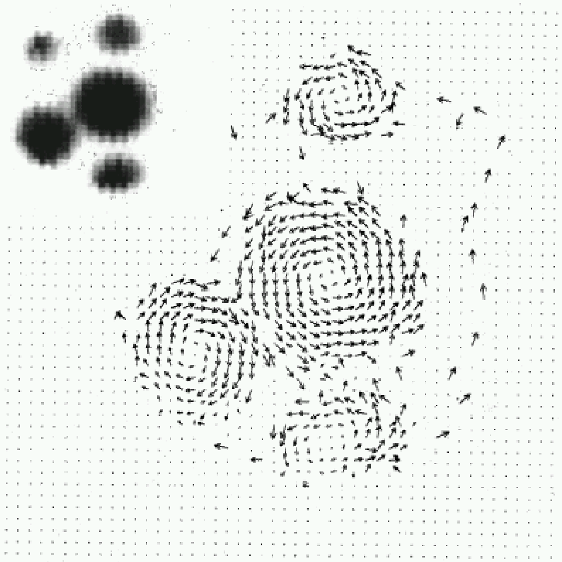


FIG. 8. A typical result of the chemoregulated model for vortex formation. The positive feedback of the chemoattractant breaks the originally homogeneous density and aggregates with high density are created. The flow field is represented by arrows of a magnitude proportional with the local velocity. The inset shows the concentration distribution of the chemoattractant ($\mu=0.1$, $\nu=0.1$, $F=0.3$, $\kappa=0.1$, $\chi_A=0.2$, $\eta=0.2$, $D_A=0.1$, $\lambda_A=0.01$).

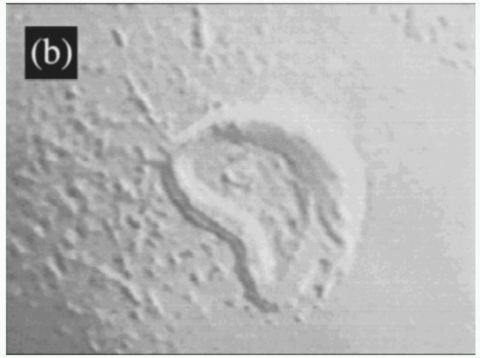
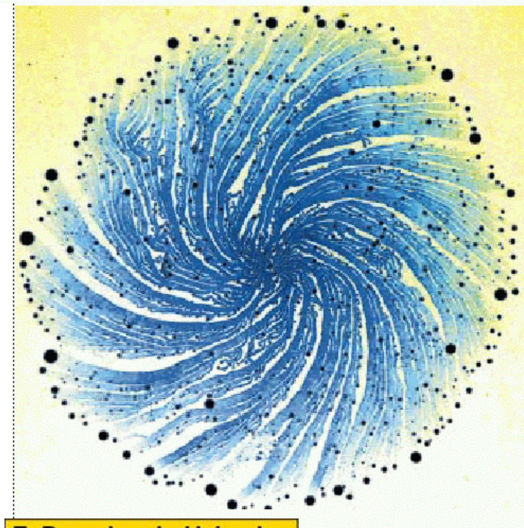


FIG. 9. In the same model as shown in Fig. 8, but for a different value of the parameter μ (providing stronger velocity-velocity interaction, $\mu=0.3$), rotating rings develop in the simulations (a). This phenomenon was also reported in Ref. [19] (b).



E. Ben–Jacob, H. Levine
***Nature*, 409**, pp. 985–986

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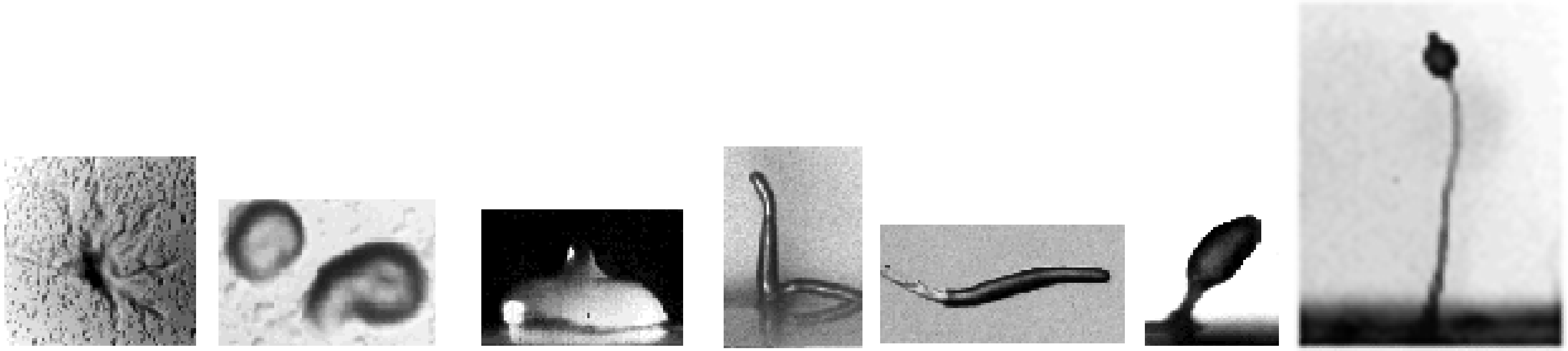
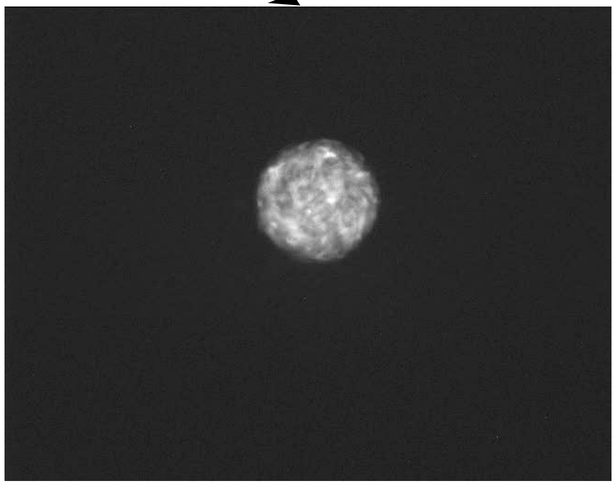
Dictyostelium discoidium Slime Molds



8 hrs 10 hrs 12 hrs 14 hrs 16 hrs 20 hrs 24 hrs

http://www.dictybase.org



Herbert Levine, UCSD

http://www.dictybase.org

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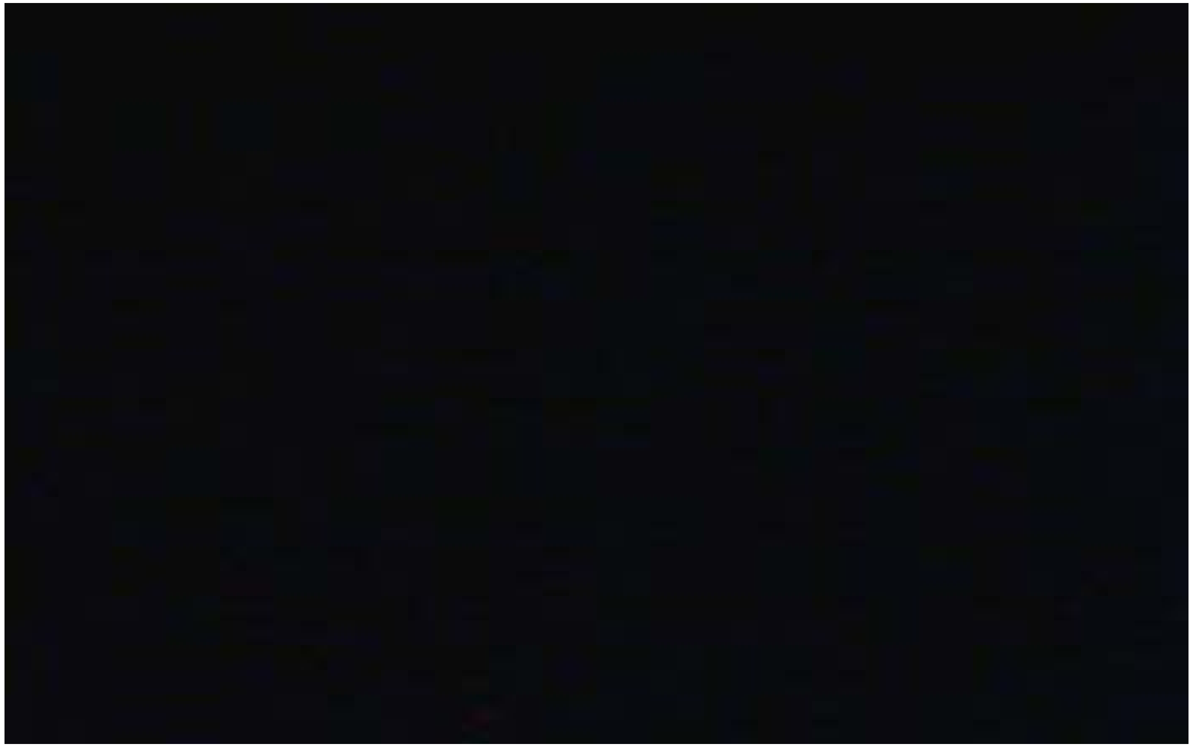
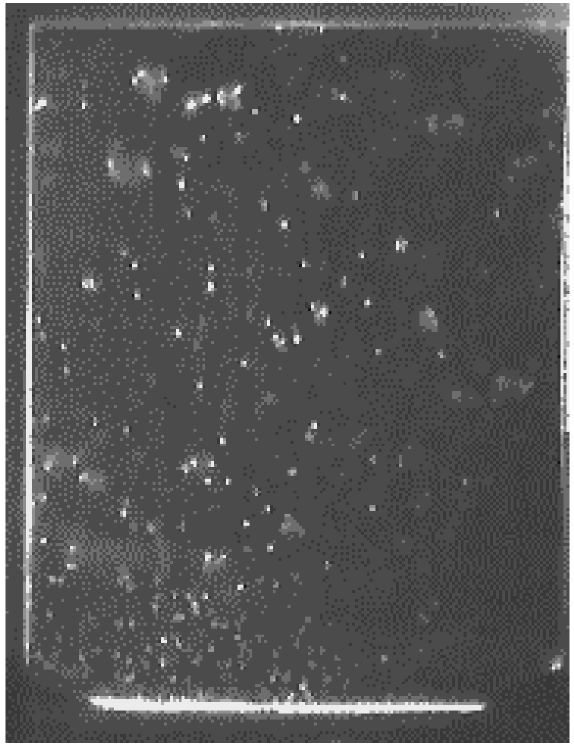
Daphnia within a Light Shaft



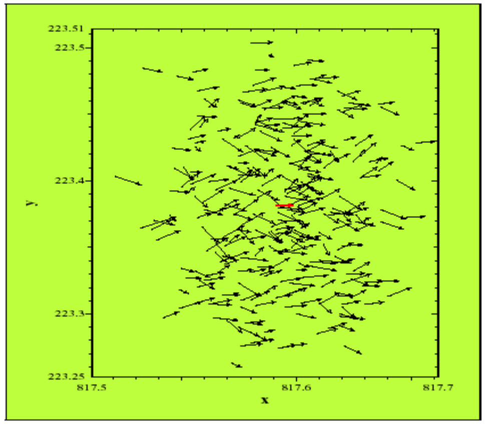
Anke Ordemann, Frank Moss:
Center for Neurodynamics, UMSL



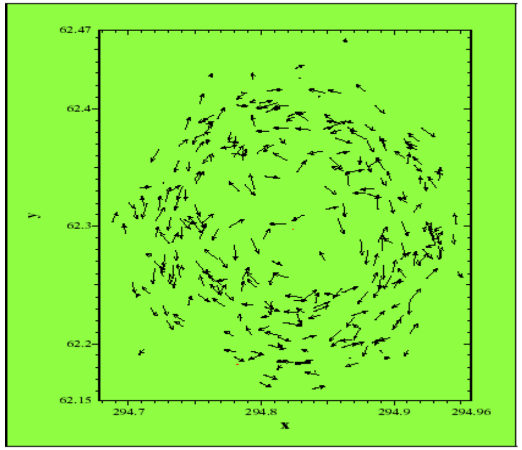
J. Rudi Strickler, Akira Okubo

Basic Observed Motions

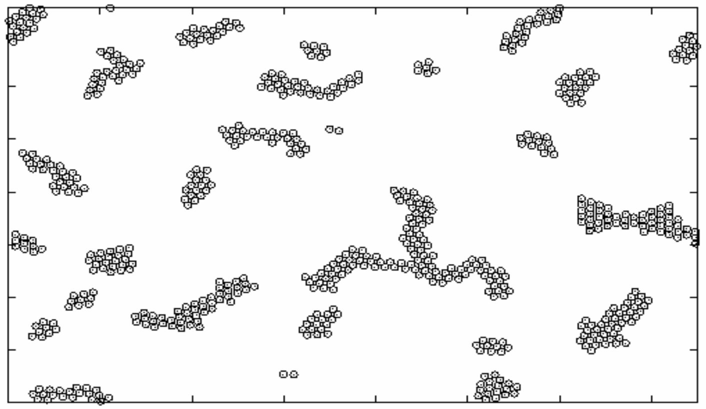
- Directed motion



- Rotational motion



- Amoebae like motion



A. Okubo & S. A. Levin: *Diffusion and Ecological Problems*, Springer, New York, 2nd edition (2003)

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Question and Answer

Question

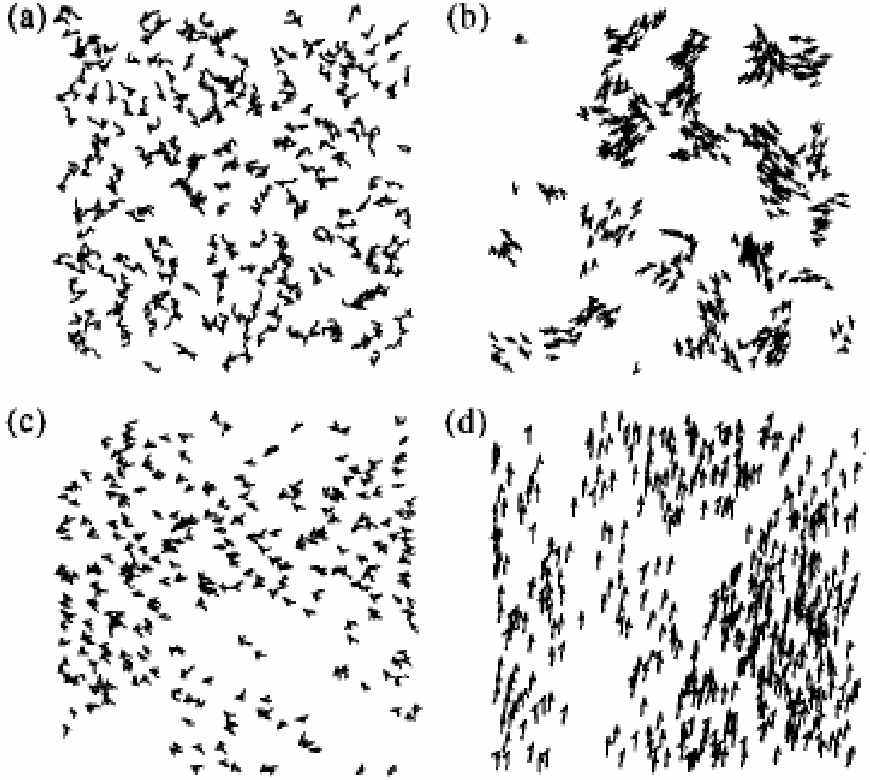
What are the basic features which have to be put into a model to resemble coherent motion as they can be observed in nature and society

Answer

One needs a model:

- which stationary state is far from equilibrium,
- where interaction of the individuals leads to a specific confinement and
- fluctuating forces

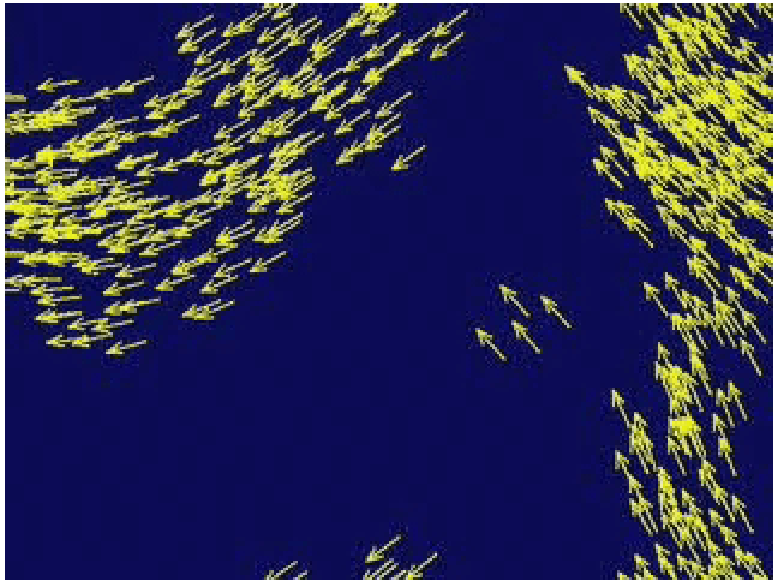
Vicsek's *XY*-Model



- N locally aligning particles with noise and constant velocity v_0

$$\vartheta_{\mathrm{i}}(t+\Delta t) = \langle \vartheta(i) \rangle_{\mathrm{S(i)}} + \xi$$

- Periodic boundary conditions
- Parameters: density of particles and amplitude of noise



$$\xi = -\frac{}{2}, \frac{}{2}$$

(a) initial random setting

(b) low density, low noise

(c) high density, high noise

(d) high density, low noise

Vicsek et al., Phys. Rev. Lett. **75**, 1226-1229 (1995)

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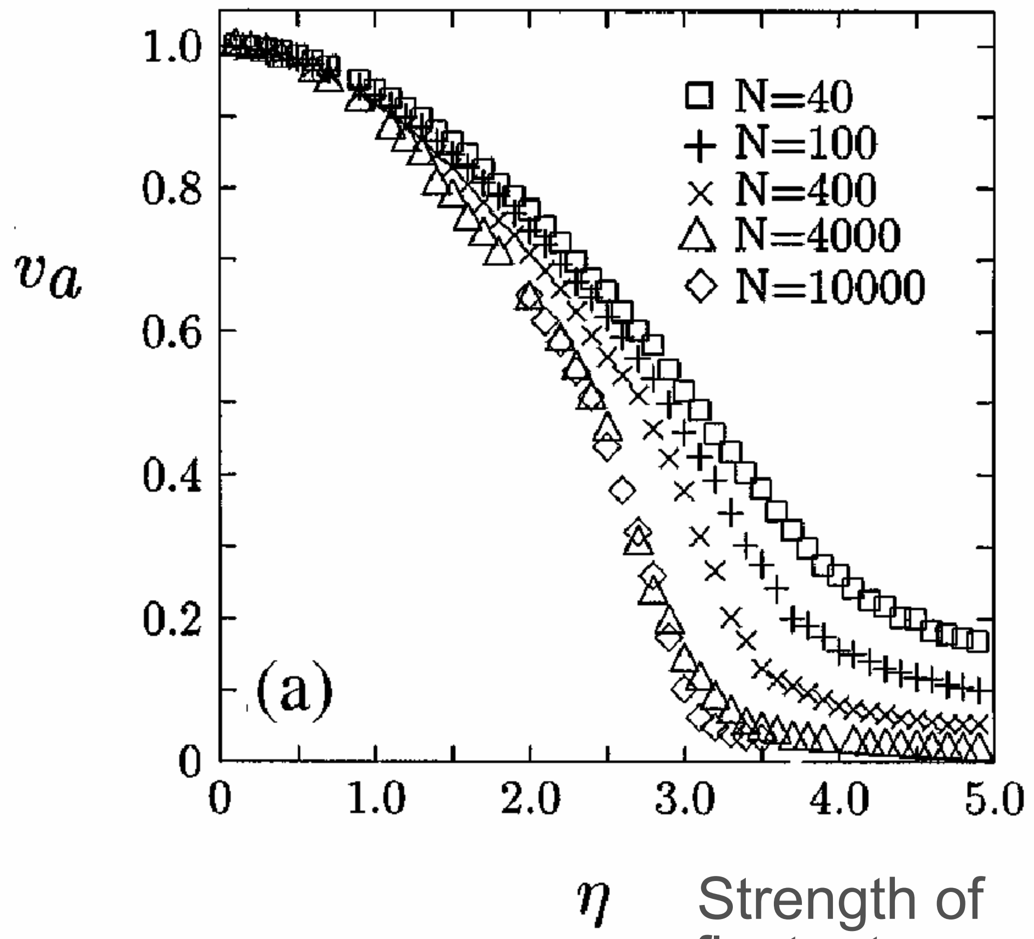
Novel Type of Phase Transition

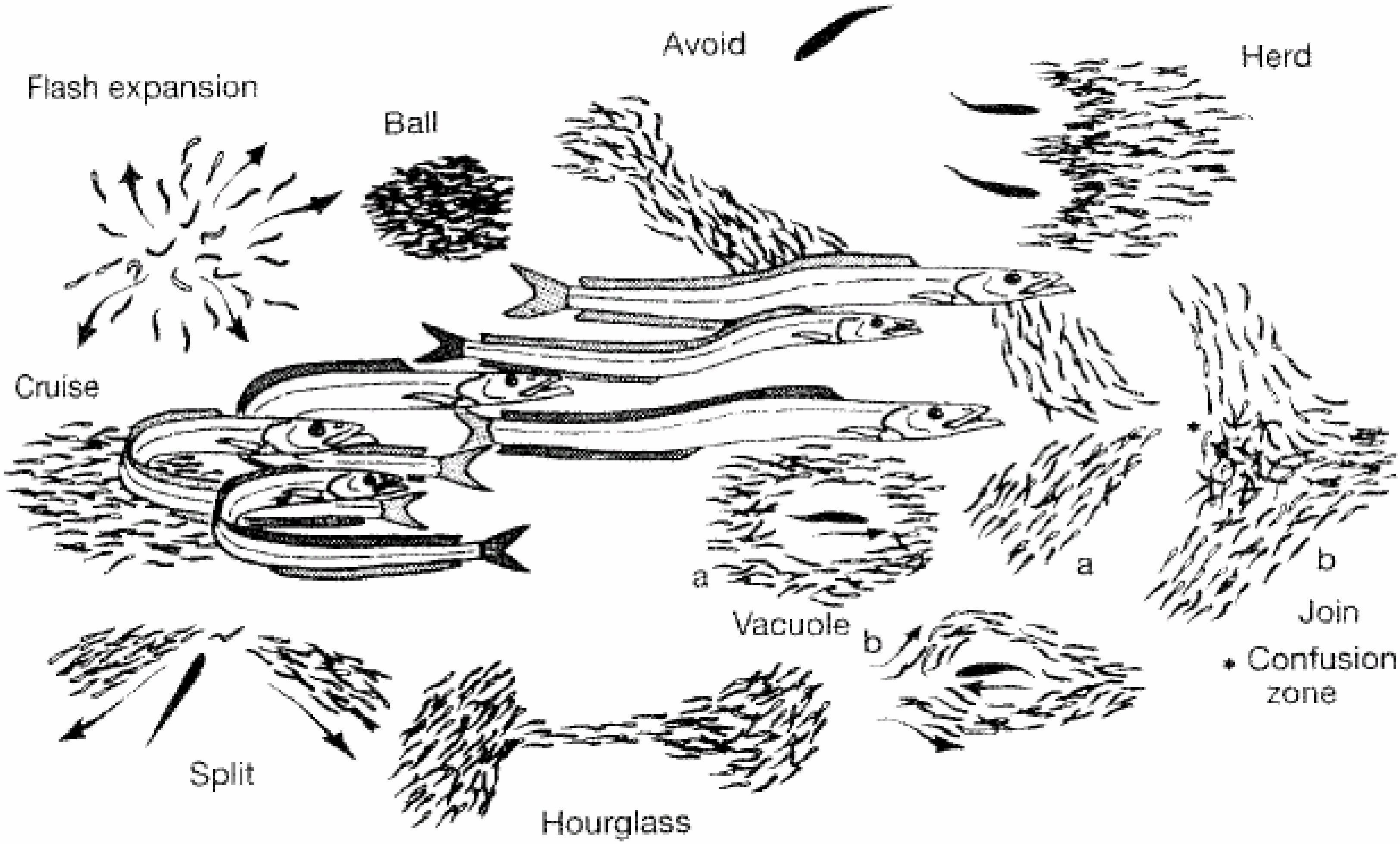
average velocity



Strength of fluctuatuons

Vicsek et al., Phys. Rev. Lett. **75**, 1226-1229 (1995)

Modelling of Fishschools



Parrish, Viscido and Grünbaum, Biological Bulletin **202**, 296-305 (2002)

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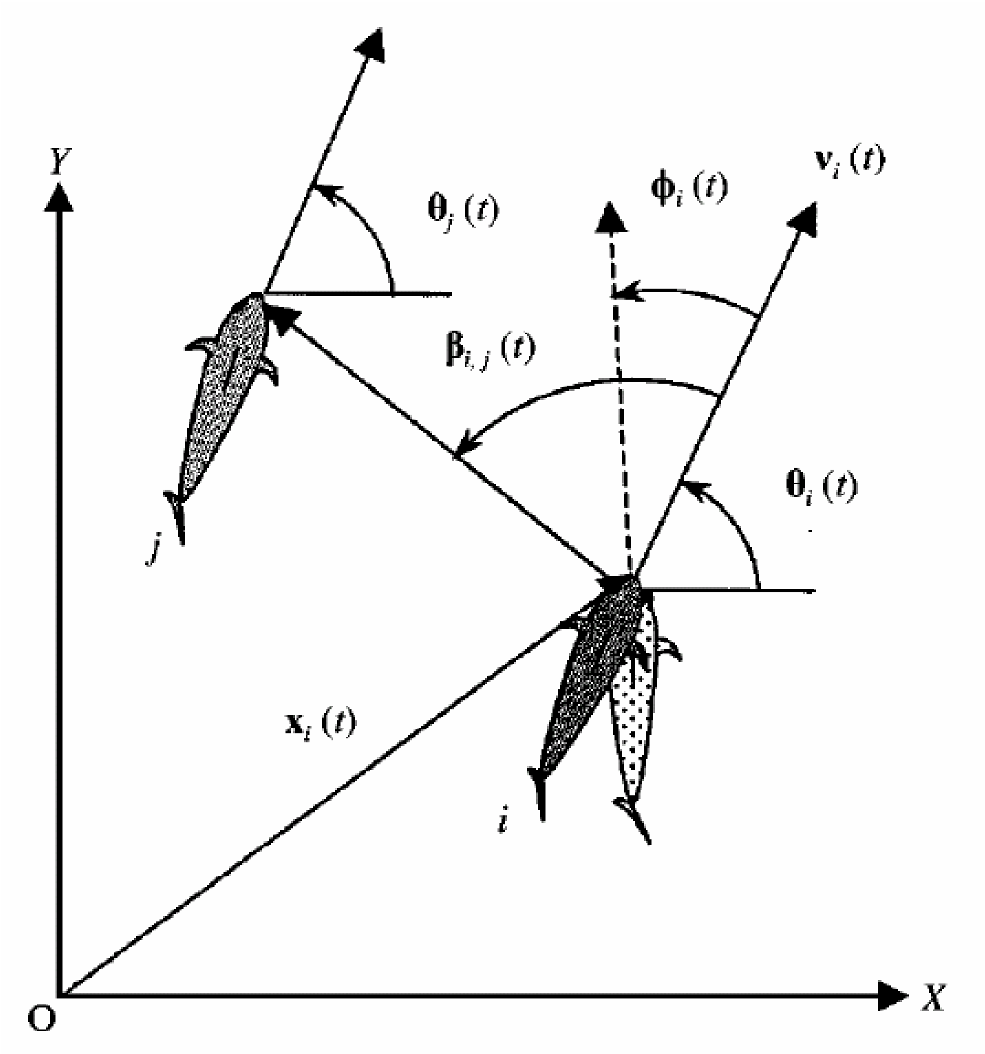
Modelling of Fishschools



$$
\begin{aligned}
\mathsf{x}_\mathsf{i}(t+1) &= \mathsf{x}_\mathsf{i}(t) + \mathsf{v}_\mathsf{i}(t)\Delta t \\
\mathsf{v}_\mathsf{i}(t) &= \{v_\mathsf{i}(t), \theta_\mathsf{i}(t)\} \\
\theta_\mathsf{i}(t) &= \theta_\mathsf{i}(t-1) + \phi_\mathsf{i}(t)
\end{aligned}
$$

Similiar to Viscek‘s model + some rules for the update of the turning angle ϕ_i

Inada and Kawachi, Journal of theoretical Biology **214**, 371-387 (2002)

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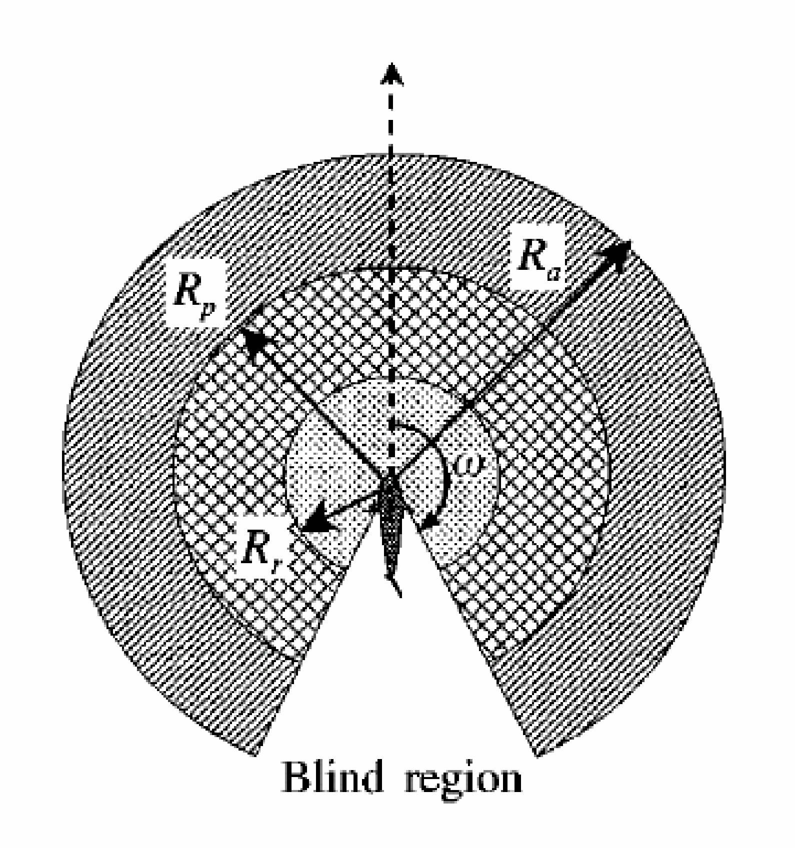
Modelling of Fishschools



Reaction field around an individual, consisting of repulsive-orientation, parallel-orientation, and attractive orientation fields whose radii are R_r , R_p, and R_a, respectively. The region beyond the attractive-orientation field is outside the detection region of an individual, and a blind region exists behind an individual because of its body.

Turning Angle Distribution

$$p(\phi_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\phi_i - \alpha_i)^2}{2\sigma^2} \right)$$

Deterministic turning angle α_i (to avoid the predator)

$$\alpha_i = \measuredangle(\mathbf{a}_i, \mathbf{v}_i(t)),$$

$$\mathbf{a}_i = c\mathbf{a}_{i,school} + (1-c)\mathbf{a}_{i,predator} \quad (0 \leqslant c \leqslant 1)$$

Inada and Kawachi, Journal of theoretical Biology **214**, 371-387 (2002)

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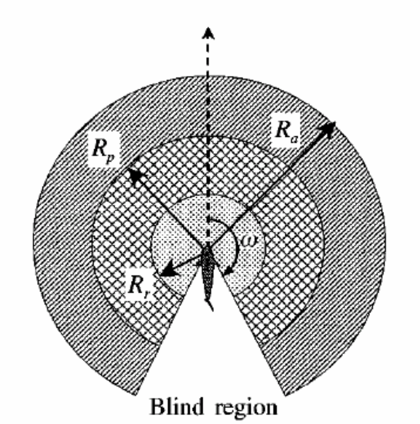
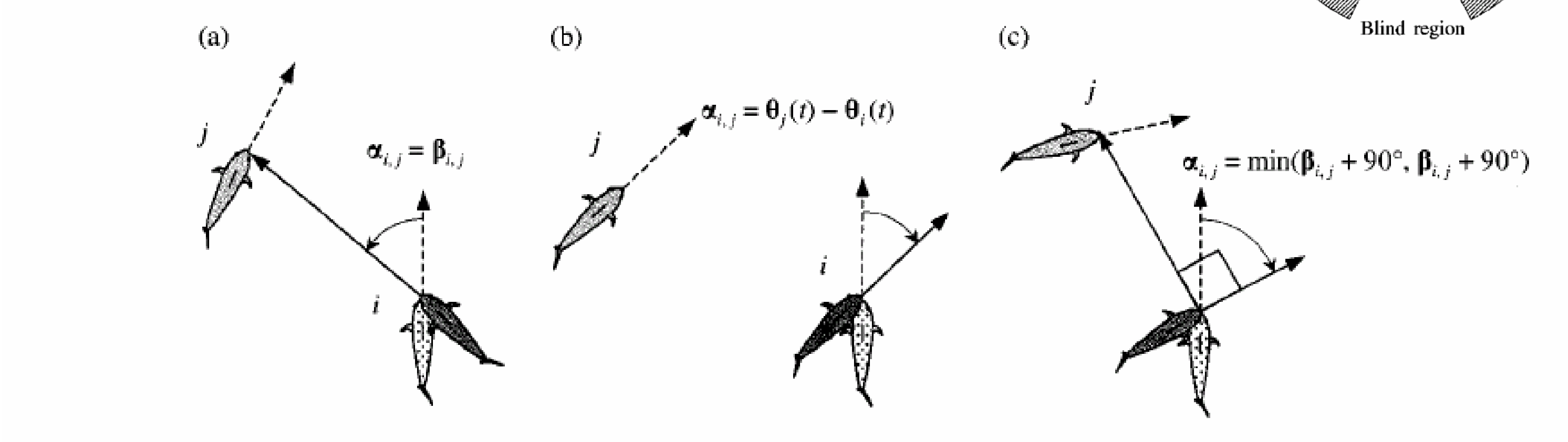
Modelling of Fishschools

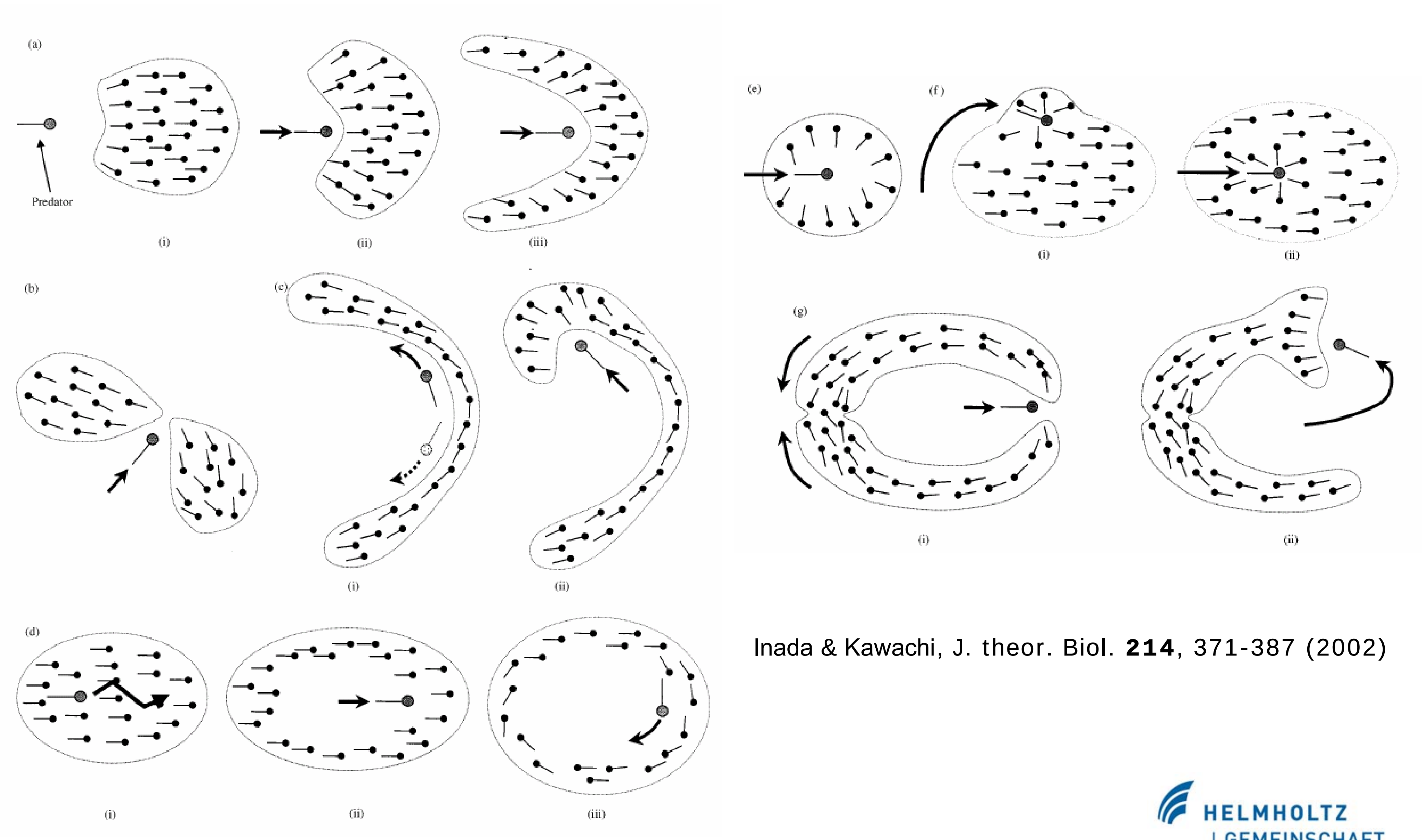




Behavioral rules for interaction with other individuals; (a) **approach**, (b) **parallel-orientation**, (c) **repulsion**. These rules were first proposed by Aoki (1982) and Huth & Wissel (1992), together with the random direction at which an individual turns to search for other individuals.

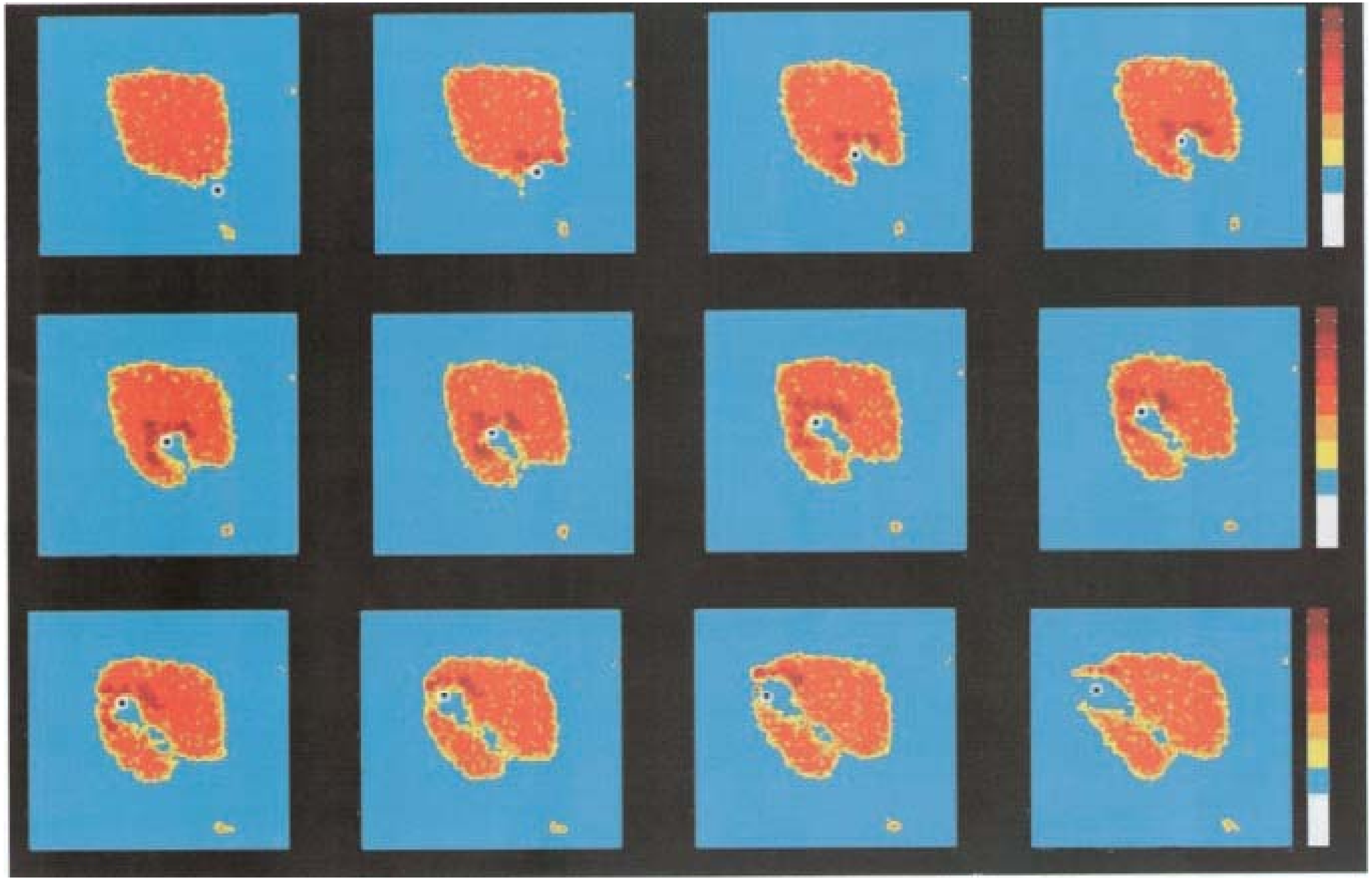
Inada and Kawachi, Journal of theoretical Biology **214**, 371-387 (2002)

Modelling of Fishschools



Inada & Kawachi, J. theor. Biol. **214**, 371-387 (2002)

Modelling of Fishschools



Parrish & Edelstein-Keshet, Science **284**, 99-101 (1999)

(Second) Newton‘s Law

Make it as simple as it is!

$$\underset{\text{force}}{F} = \underset{\text{mass}}{m} \times \underset{\text{accelaration}}{a}$$

change of velocity per unit time

$$a = \frac{\Delta v}{\Delta t} = \frac{1}{\Delta t}\left(v_{t+\Delta t} - v_t\right) \overset{\Delta t \to 0}{=} \frac{dv}{dt} = \dot{v}$$

$$v = \frac{\Delta s}{\Delta t} = \frac{1}{\Delta t}\left(s_{t+\Delta t} - s_t\right) \overset{\Delta t \to 0}{=} \frac{ds}{dt} = \dot{s}$$

traveled distance per unit time

From now on the distance ***s*** is denoted by the change of the space coordinates ***x*** (1D) or ***r*** (3D)

Active Brownian Particles

$$ma = F = F_{\text{depot}} + F_{\text{interaction}} + F_{\circ\,\text{uctuation}}$$

$$\dot{\boldsymbol{r}} = \boldsymbol{v}\,, \qquad m\dot{\boldsymbol{v}} = -\gamma(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{F}(\boldsymbol{r}) + \sqrt{2D}\boldsymbol{\xi}(t)$$

- Friction term (nonlinear dependance on v)
- Confinement (external boundary conditions or interaction with other particles)
- Random forces (Gaussian white noise)

Numerical Implementation

$$
\begin{aligned}
\dot{x} &= \frac{\mathrm{d}x}{\mathrm{d}t} \rightarrow \frac{\Delta x}{\Delta t} \\
\dot{v} &= \frac{\mathrm{d}v}{\mathrm{d}t} \rightarrow \frac{\Delta v}{\Delta t} \\
x(t+\Delta t) &= x(t) + v(t)\Delta t \\
v(t+\Delta t) &= v(t) + [\gamma(v(t))v(t) + F(x(t))]\,\Delta t \\
&\quad + \xi(t)^{\mathsf{p}}\,\overline{\Delta t)}
\end{aligned}
$$

Depot Model

Particle with mass m, position $\boldsymbol{r}$, velocity $\boldsymbol{v}$, self-propelling force connected to energy storage depot $e(t)$; velocity dependent friction $\gamma(\boldsymbol{v})$

External parabolic potential $U(\boldsymbol{r})$ and noise $\xi(t)$

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ m\dot{\mathbf{v}} &= d_2 e(t)\mathbf{v} - \gamma_0 \mathbf{v} - \nabla U + \text{»}(t) \end{aligned}$$

Energy depot: space-dependent take-up $q(\boldsymbol{r})$, internal dissipation $ce(t)$, conversion of internal energy into kinetic energy $d_2\, e(t)\, v^2$

$$\dot{e}(t) = q(\mathbf{r}\ ) - ce(t) - d_2 e(t)\mathbf{v}^2$$

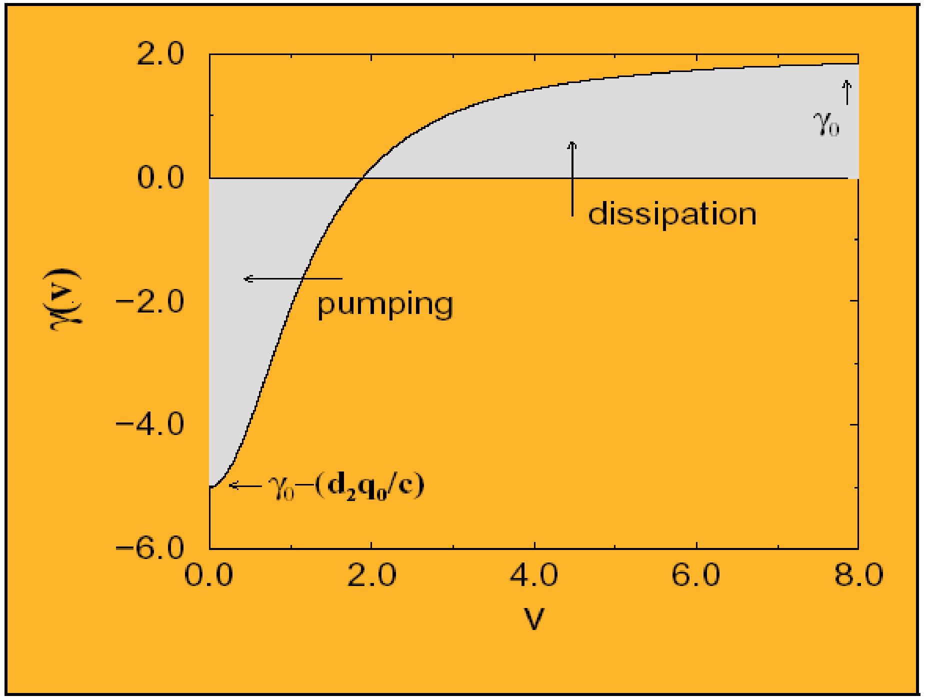
Energy depot analysis (for $q(\boldsymbol{r}) = q_0$):

$$\gamma(\mathbf{v}) = \gamma_0 - \frac{d_2 q_0}{c_0 + d_2 \mathbf{v}^2}$$

Ebeling et al., Biosystems **49**, 17-29 (1999)

Negative Friction

$$\gamma(\boldsymbol{v}) = \gamma_0 - \frac{q_0\, d_2}{c + d_2 \boldsymbol{v}^2}$$



EBELING et. al, Biosystems **49**, 17–29 (1999)

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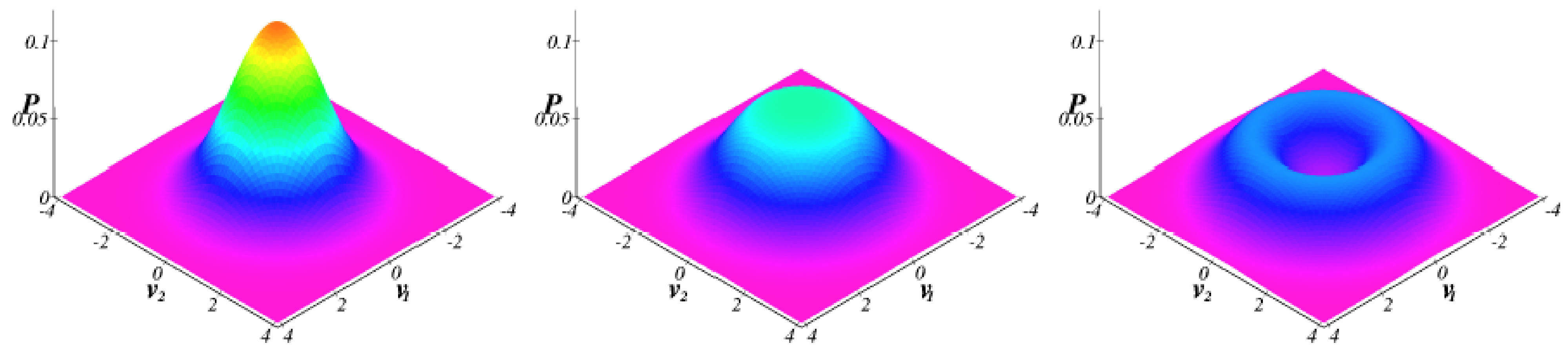
Depot model: $\gamma(\boldsymbol{v}) = \gamma_0 - \frac{q_0\, d_2}{c + d_2 \boldsymbol{v}^2}$

Stationary probabilty for the velocitiy of a particle

$$P_0(\boldsymbol{v}) = C \left(1 + \frac{d_2}{c} \boldsymbol{v}^2\right)^{\frac{q_0}{2D}} \exp\left[-\frac{\gamma_0}{2D} \boldsymbol{v}^2\right]$$



$d_2 = 0.07$ $\quad$ $d_2 = 0.2$ $\quad$ $d_2 = 0.7$

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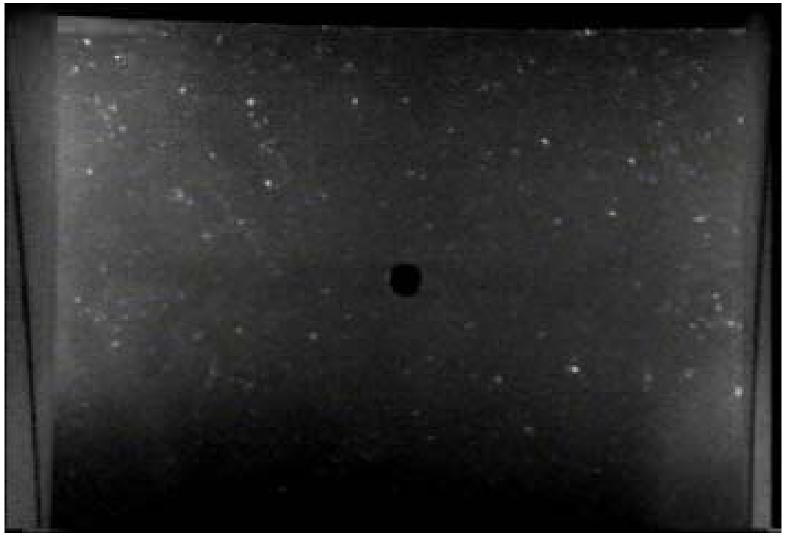
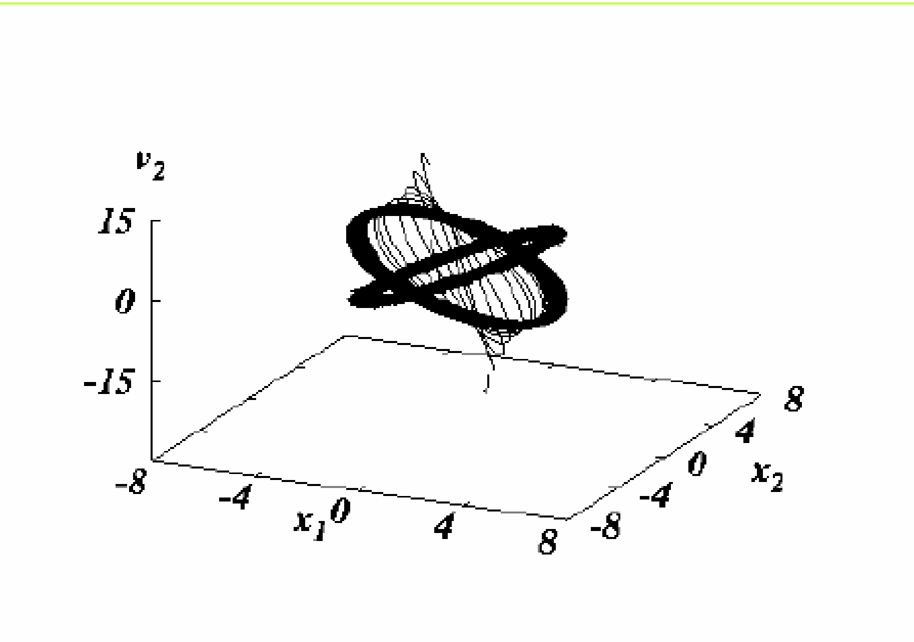
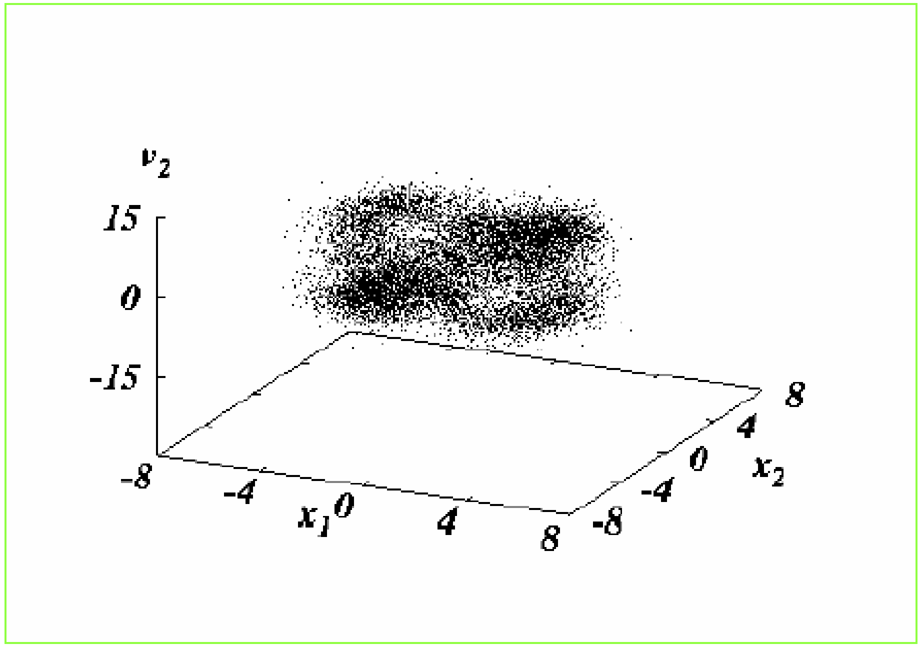
Active Particles in an External Potential





weak noise

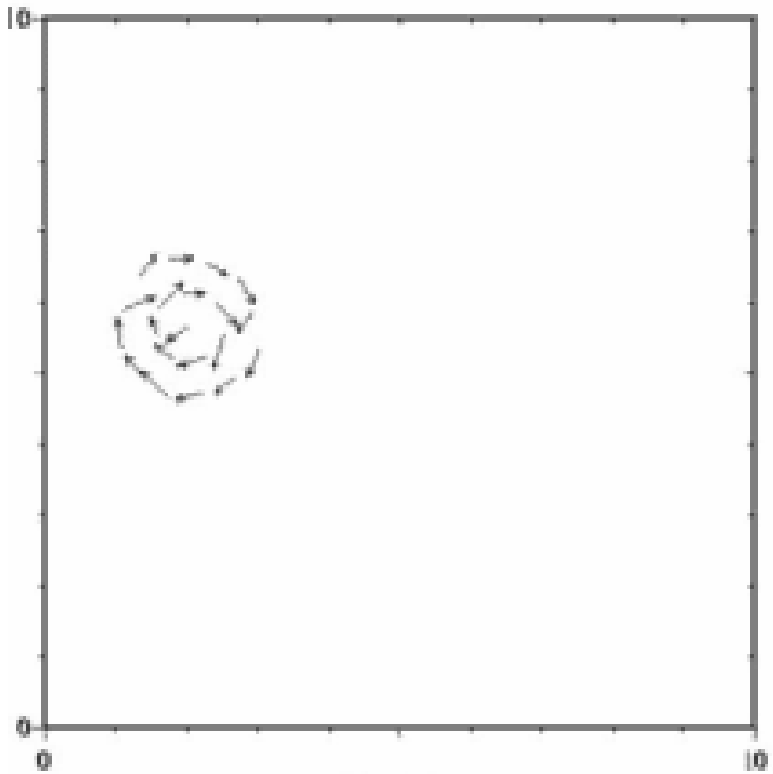


strong noise

Erdmann et al., European Physical Journal B **15**, 105-113 (2000)

Directly Interacting Particles

Herbert Levine, UCSD



Erdmann et al., Physical Review E **65**, 061106 (2002)

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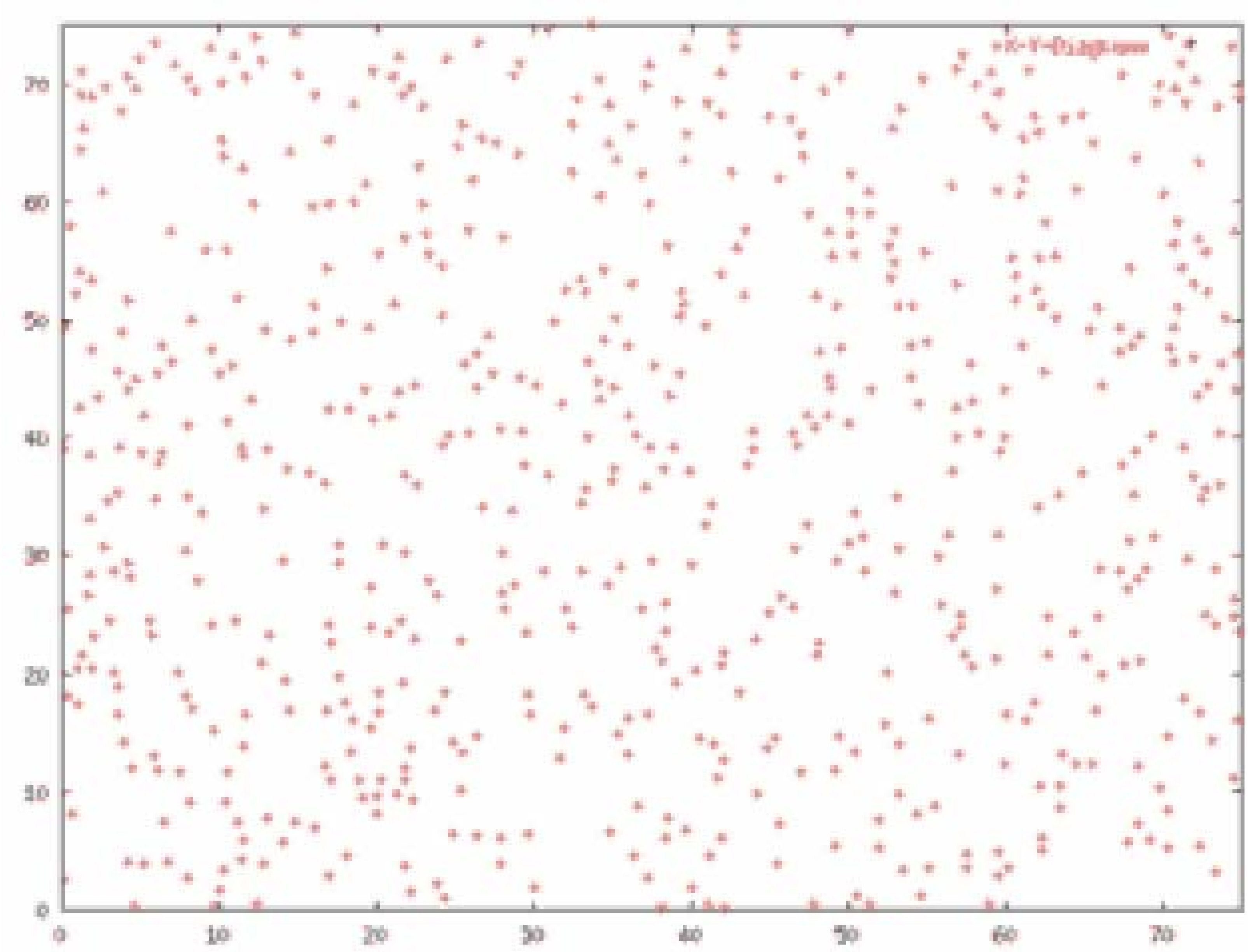

Ebeling and Erdmann, Complexity **8**(4), 23-30 (2003)

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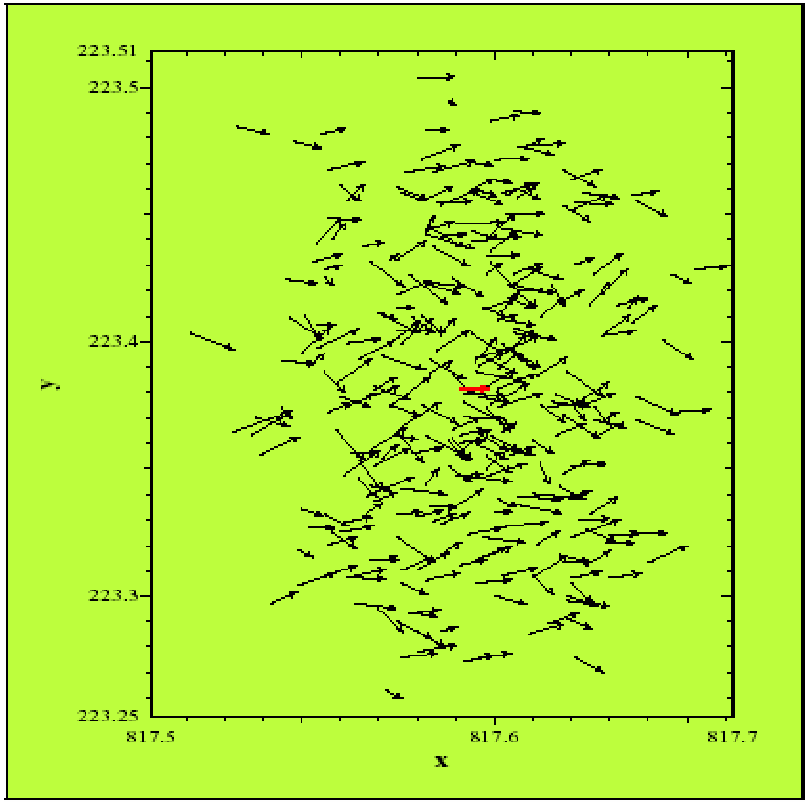
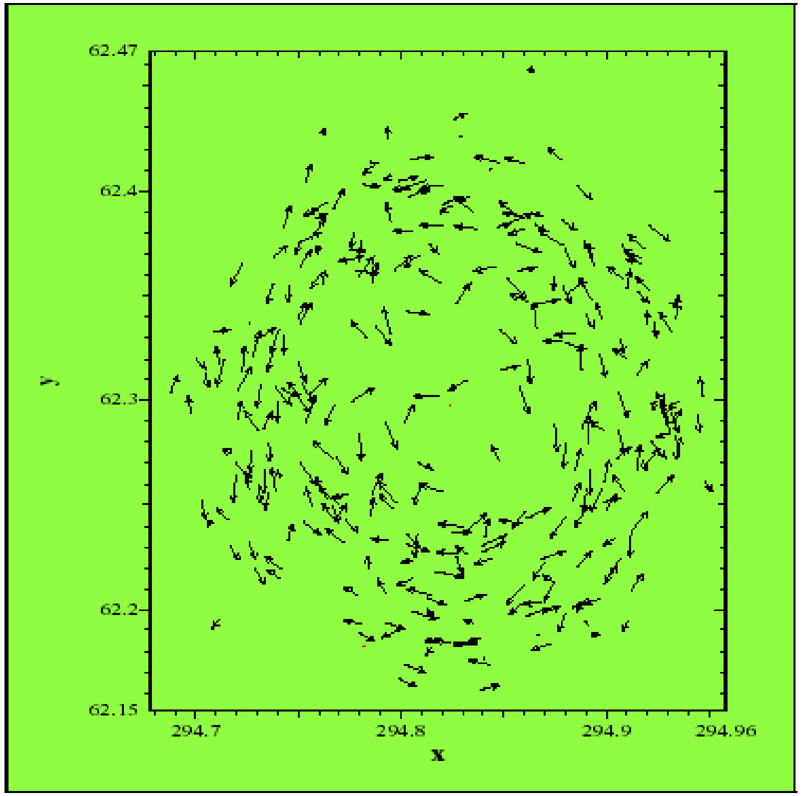
Harmonic Interaction

$$m\ddot{\boldsymbol{r}}_i = \left(\gamma_1 - \gamma_2\,\dot{\boldsymbol{r}}_i^2\right)\dot{\boldsymbol{r}} - \frac{a}{N}\sum_{j=1}^{N}\left(\boldsymbol{r}_i - \boldsymbol{r}_j\right) + \sqrt{2D}\,\boldsymbol{\xi}_i(t)$$

Translational mode



Rotational mode



Erdmann et al., Physical Review E **71**, 051904 (2005)

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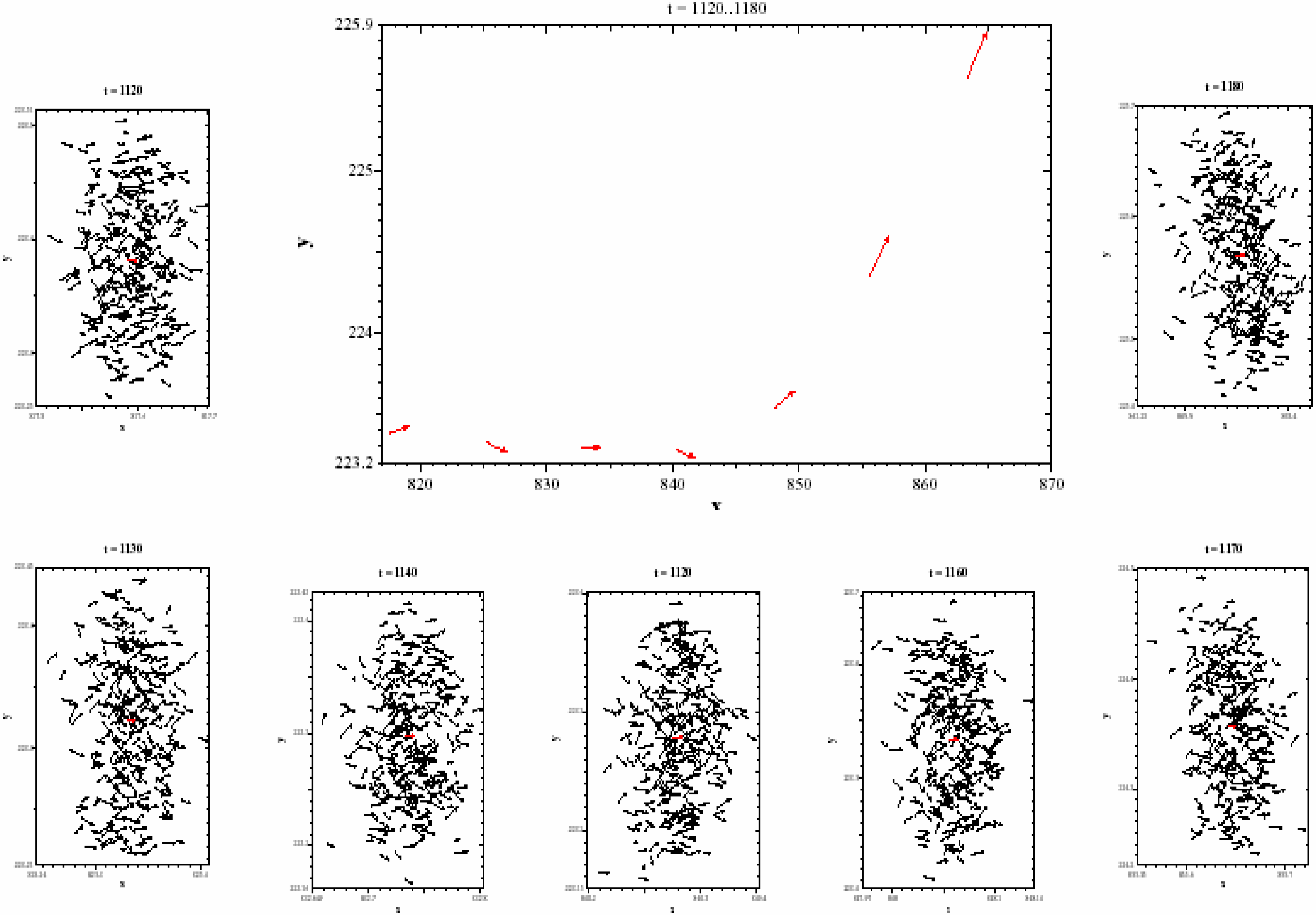
Center of Mass



Erdmann et al., Physical Review E **71**, 051904 (2005)

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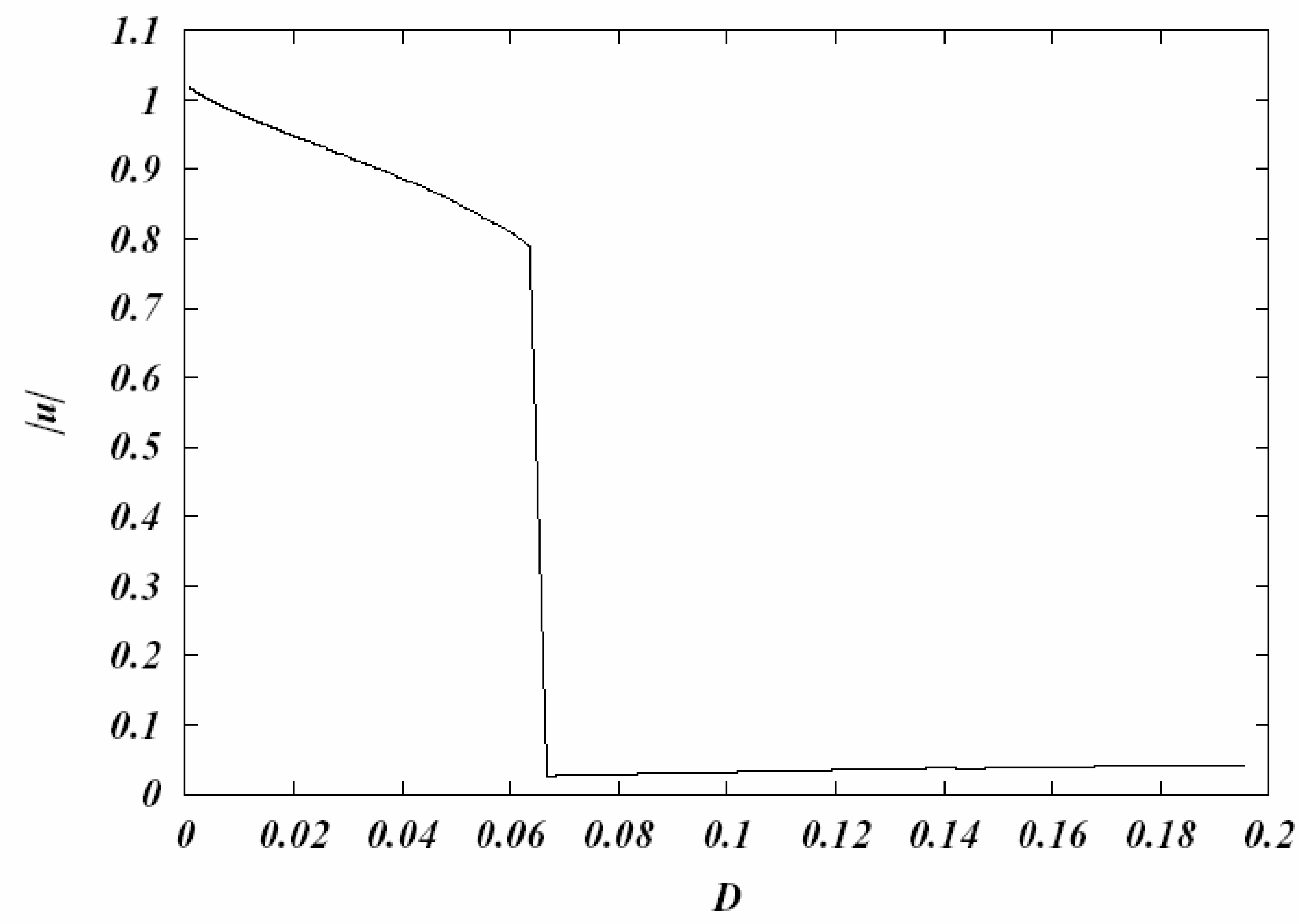
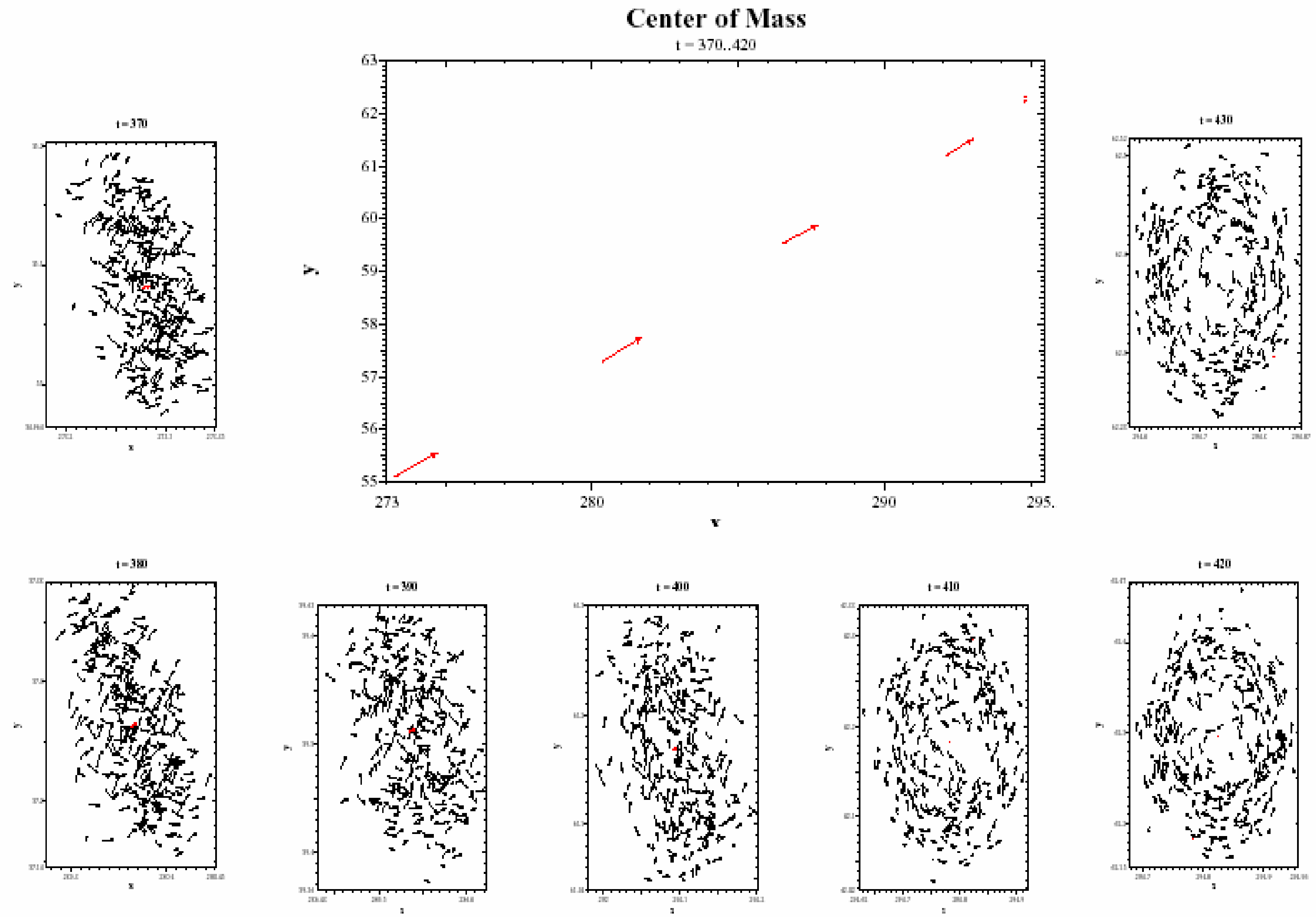
Stability of the Translational Mode



Erdmann et al., Physical Review E **71**, 051904 (2005)



Erdmann et al., Physical Review E **71**, 051904 (2005)

Active Brownian Particles with Chemical Interaction

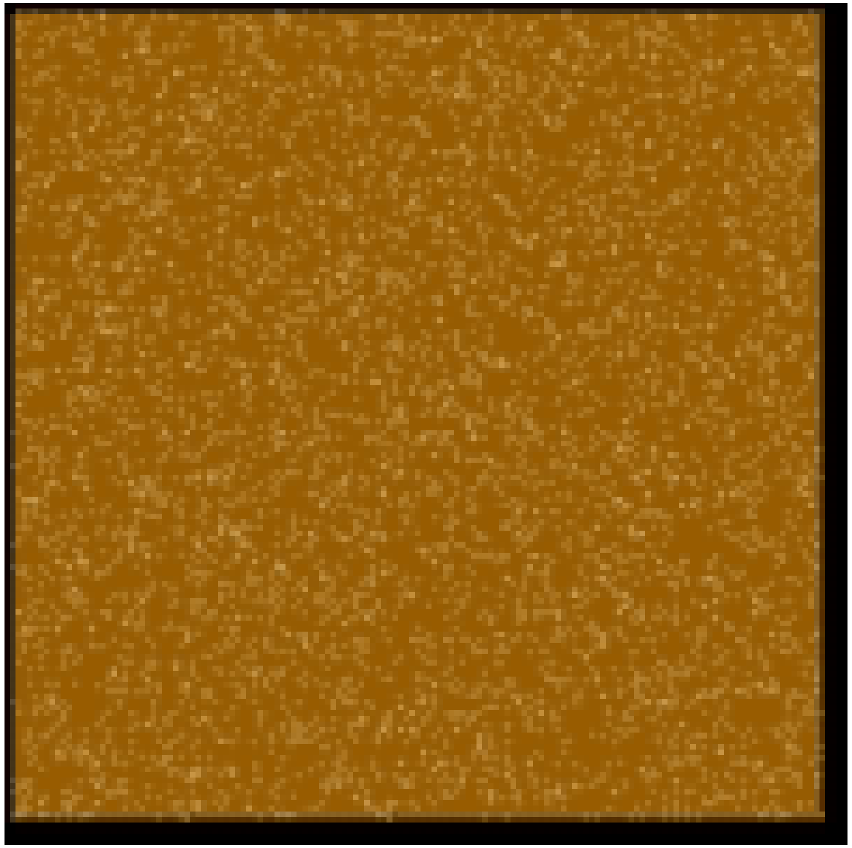
$$\dot{\mathbf{r}}_i = \mathbf{v}_i$$

$$m\dot{\mathbf{v}}_i = \underbrace{-\left(\gamma_1 - \gamma_2 \mathbf{v}_i^2\right)}_{-\gamma(\mathbf{v})} \mathbf{v}_i + \underbrace{\beta(c)\nabla c}_{\text{chemotaxis}} + \underbrace{(2D)^{\frac{1}{2}} \xi_i(t)}_{\text{gaussian noise}}$$

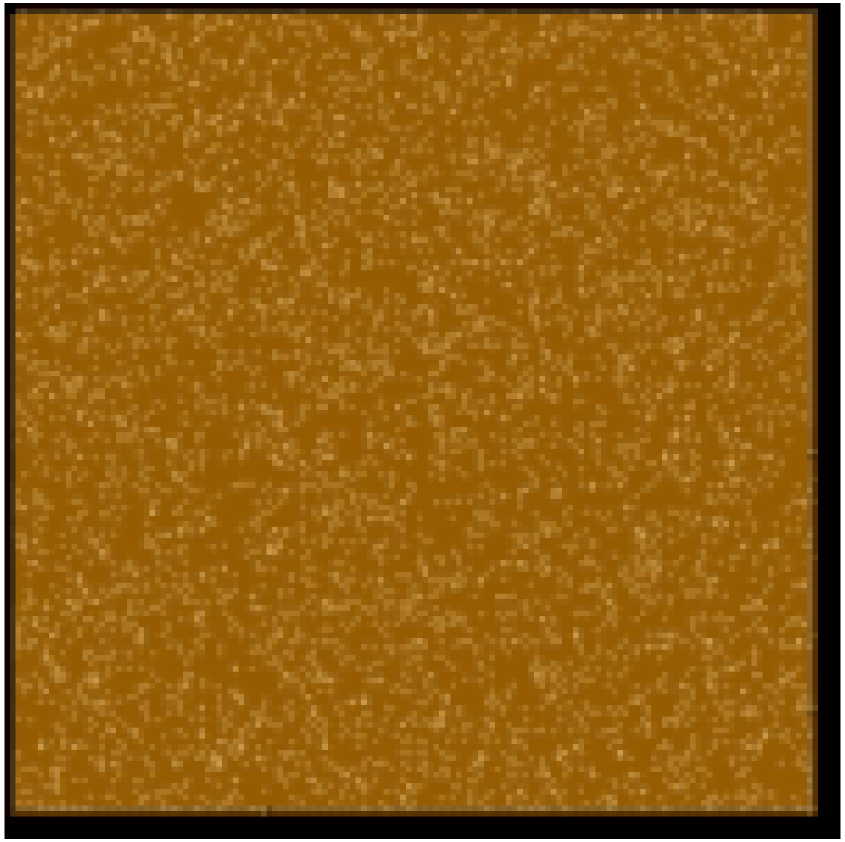
$$\dot{c} = \underbrace{\frac{q}{N}\sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i)}_{\text{production}} - \underbrace{kc}_{\text{decay}} + \underbrace{D_c \Delta c}_{\text{diffusion}}$$

$$A \xrightarrow{q} A + B$$

Cluster formation

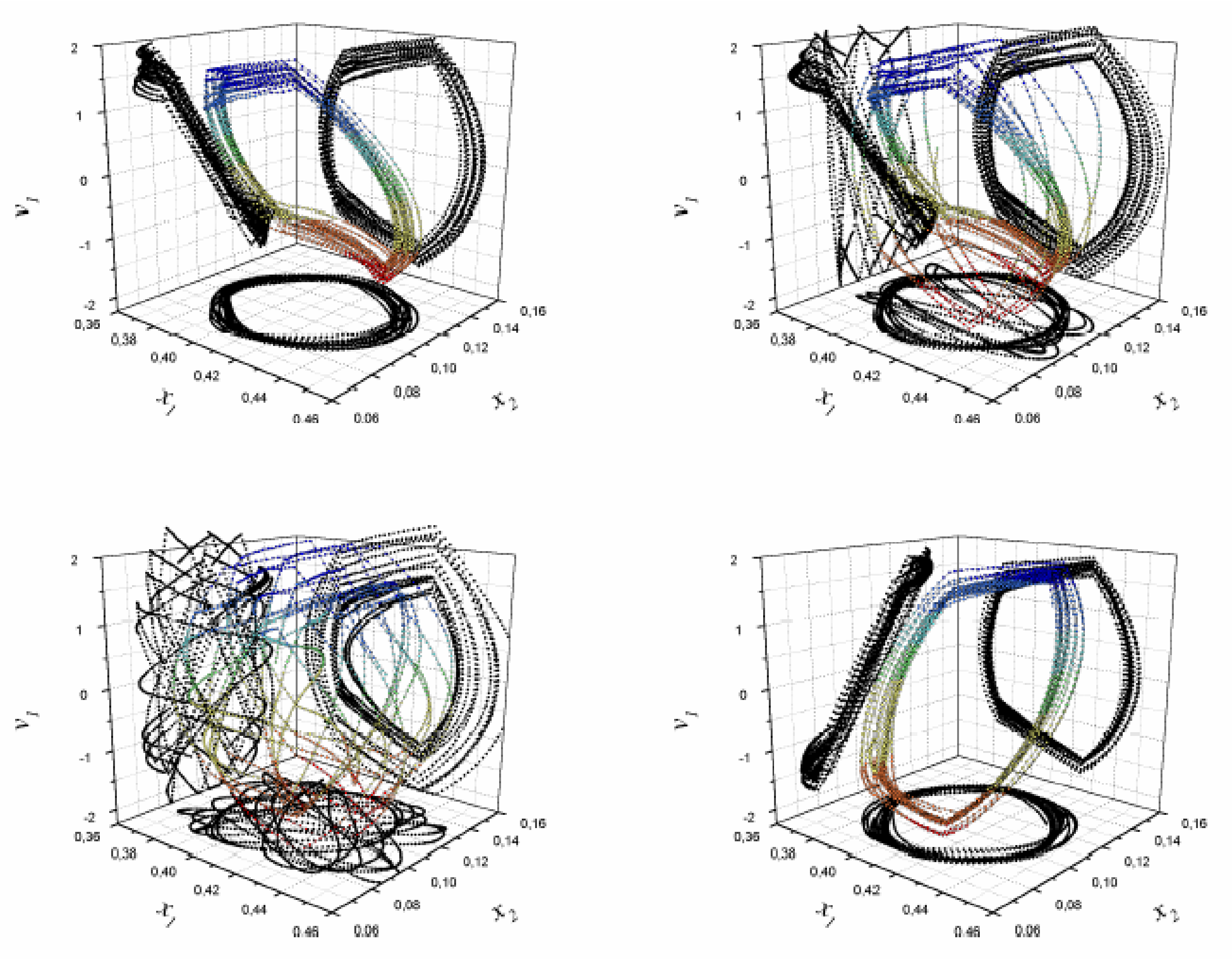


$D_c = 0.001$



$D_c = 0.1$

Trajectories of the Particles in a Cluster

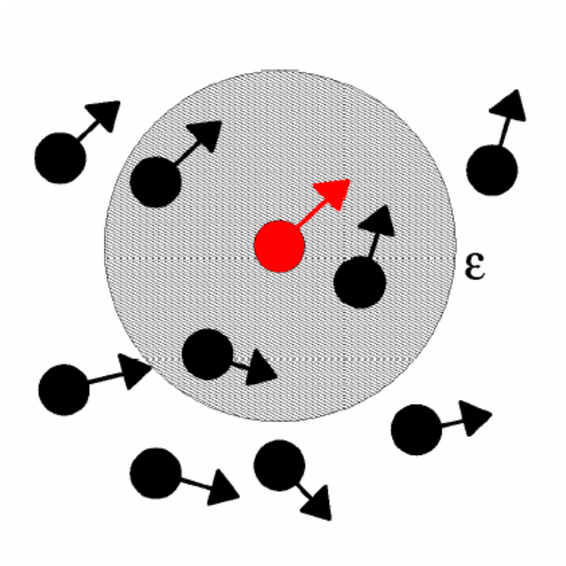


APB with fluid like interaction

$$\dot{v}_i = (\gamma_1 - \gamma_2 v_i^2)v_i + \beta(c)\nabla c + \chi v_F + (2D)^{\frac{1}{2}} \xi_i(t)$$

χ velocity-velocity interaction strength

v_F local velocity field



$$v_F = \langle v \rangle_\epsilon = \frac{1}{N_\epsilon} \sum_{j \neq i} v_j \, \delta_{r_j \in \epsilon}$$

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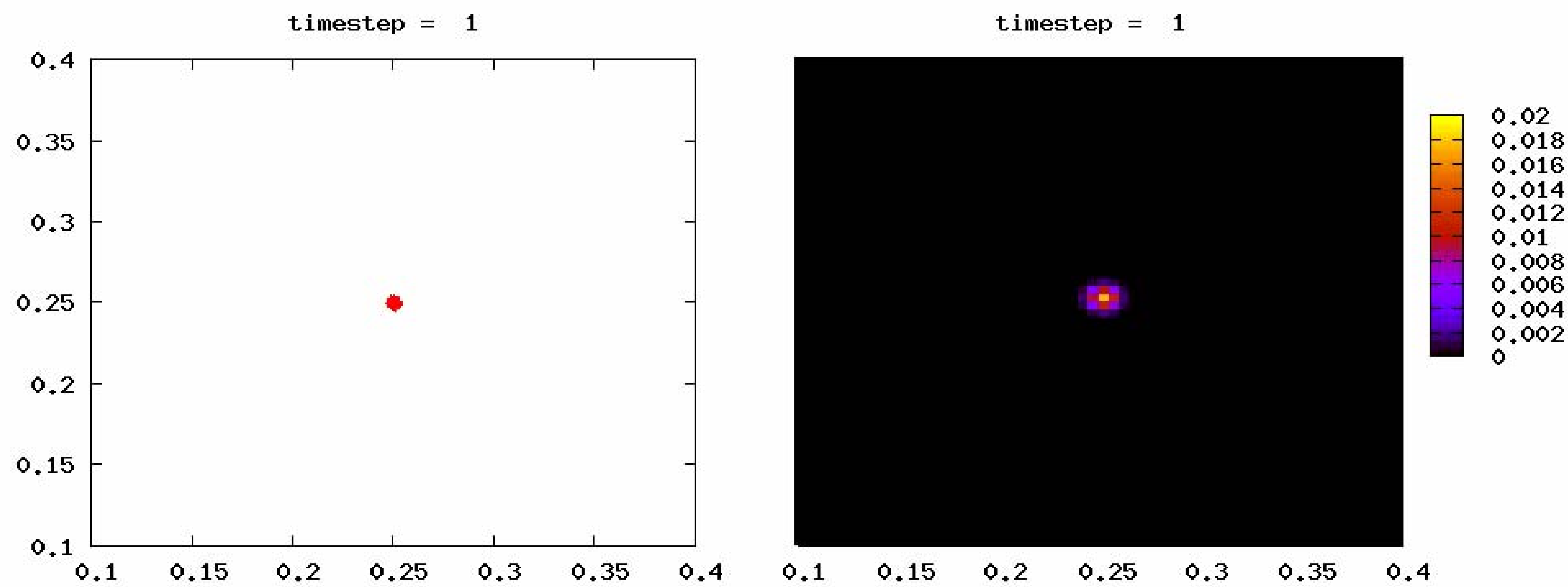
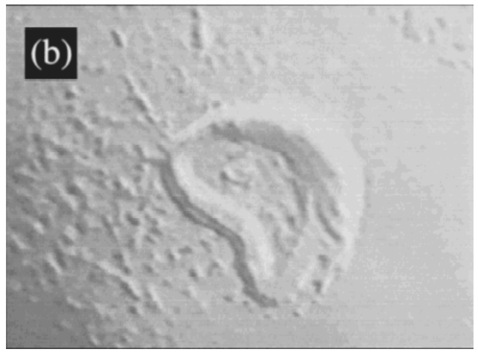
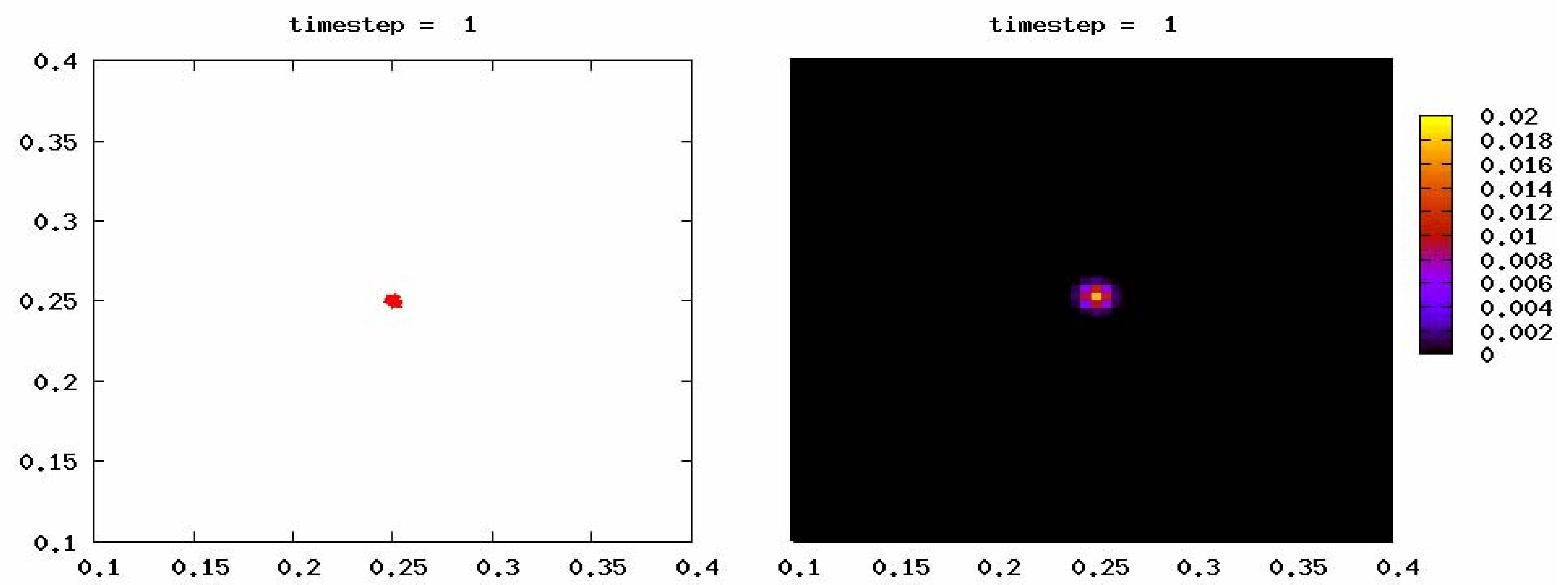
Chemical Interaction + Velocity Coupling



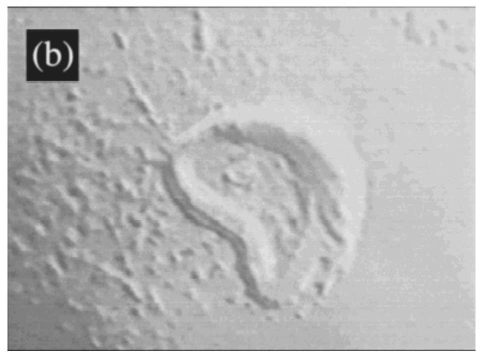


$\chi=1$

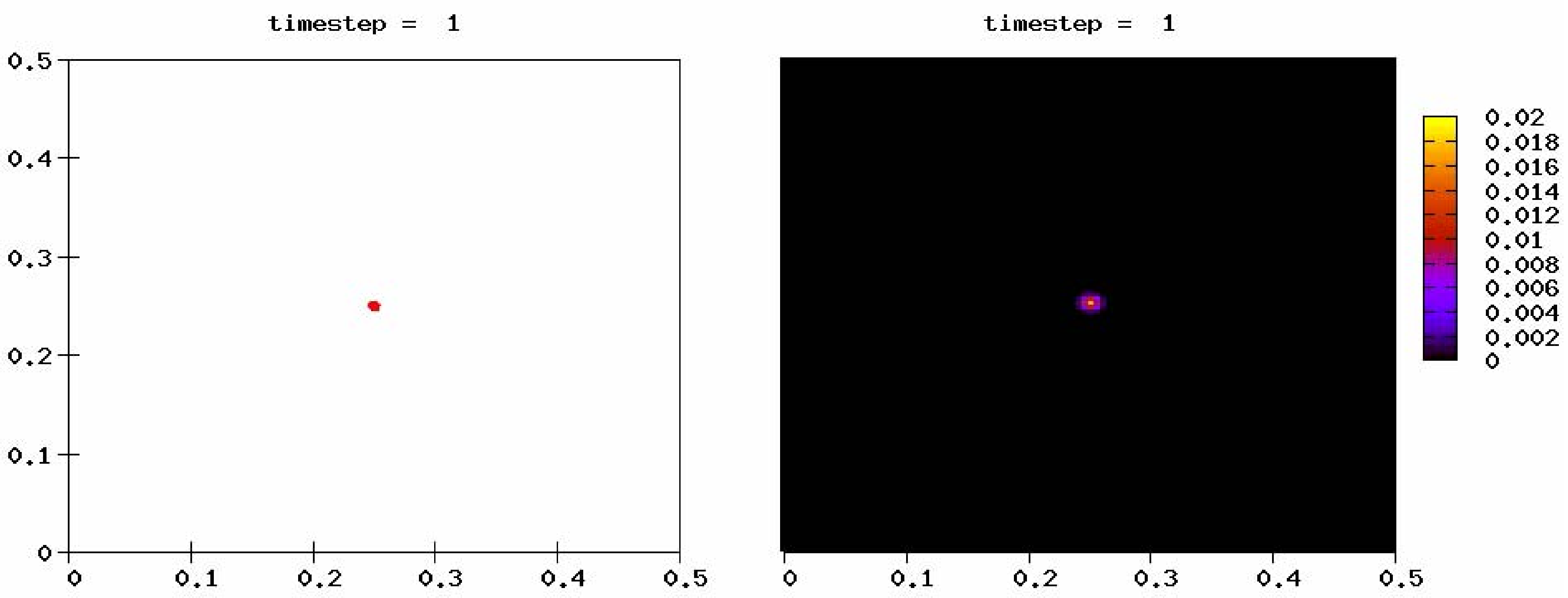
Chemical Interaction + Velocity Coupling



$\chi=3$

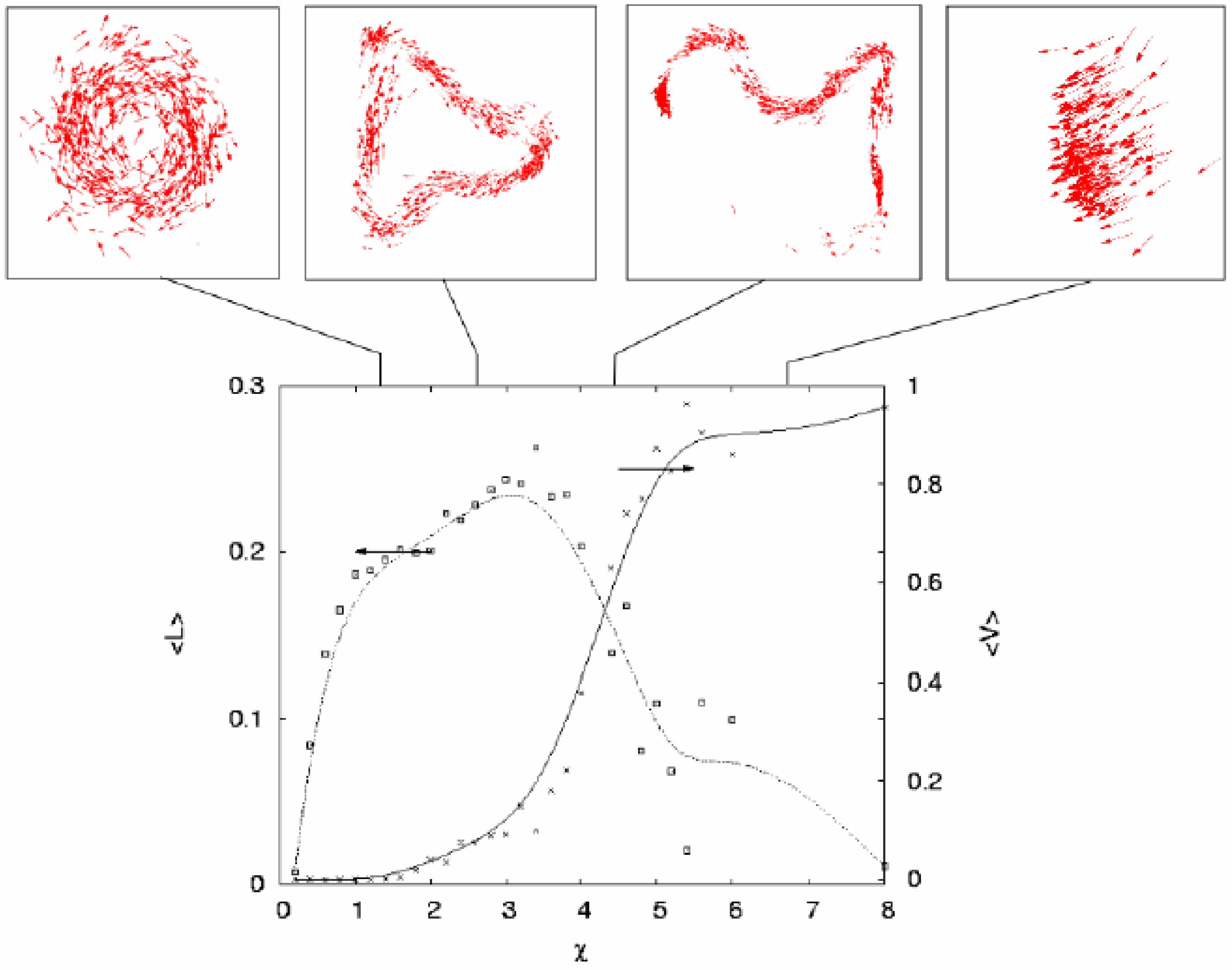


Chemical Interaction + Velocity Coupling

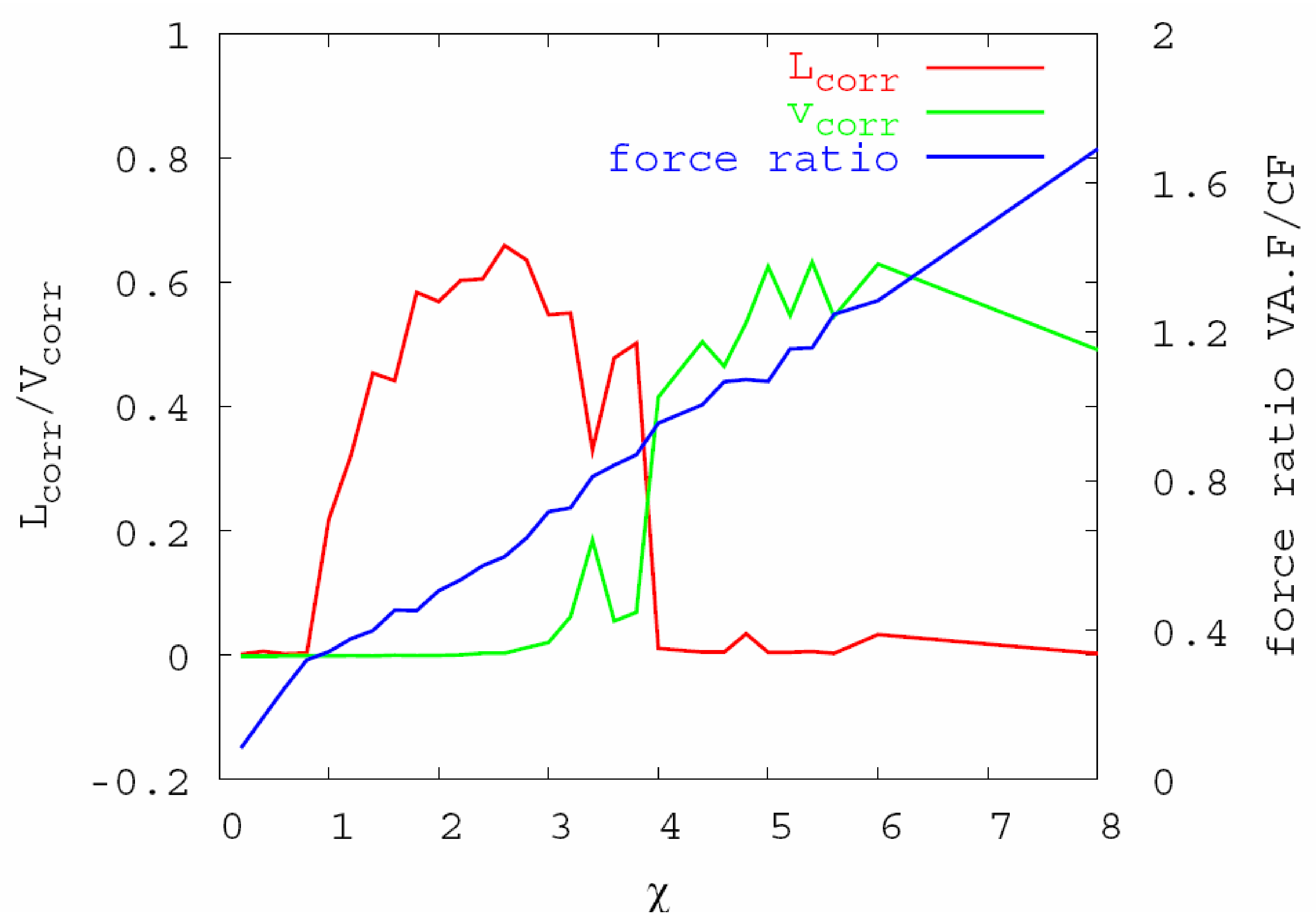


$\chi=5$

Complex χ-dependent Dynamics



Chemical Interaction + Velocity Coupling



Take-home Message

- Living far from equlibrium and
- interacting and/or
- behave non-linear

lets us live self-organized more than emergent, though we have emerged already on this planet ;-)

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Principle of Adiabatic Elimination

$$\begin{aligned}\dot{q}_1 &= -\zeta_1 q_1 - a q_1 q_2 \\ \dot{q}_2 &= -\zeta_2 q_2 + b q_1^2\end{aligned}$$

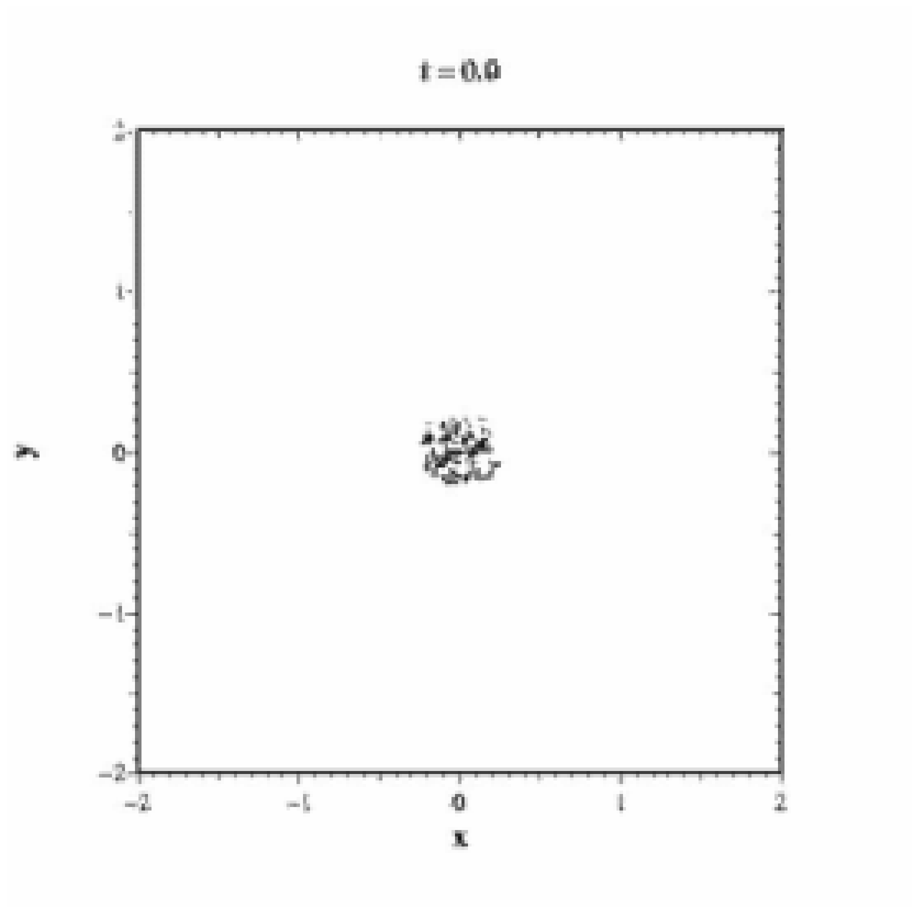
with $\zeta_2 \gg \zeta_1$

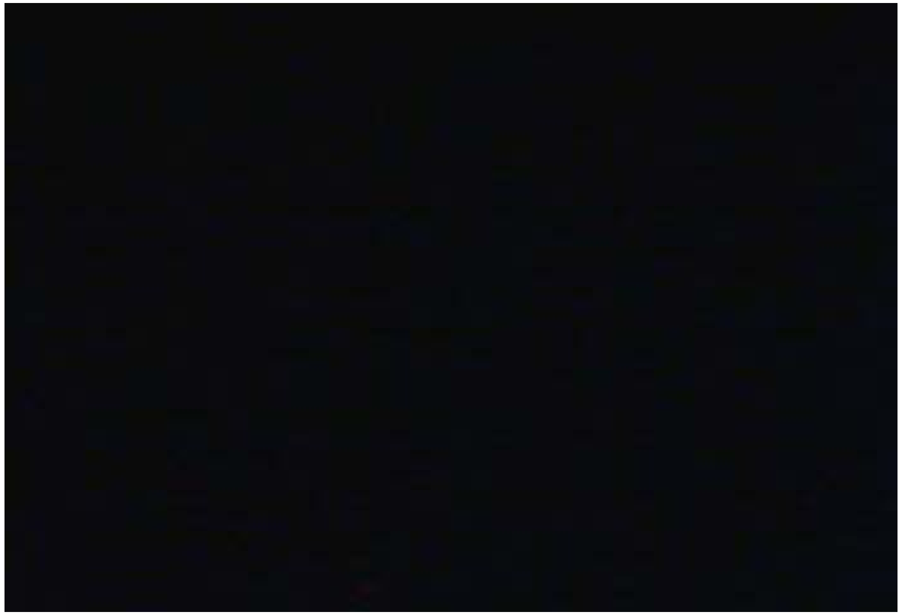
time scale separation

$$\Longrightarrow \quad \dot{q}_2 = 0 \quad \Longrightarrow \quad q_2 \approx \frac{1}{\zeta_2} b q_1^2$$

$$\Longrightarrow \quad \dot{q}_1 = -\zeta_1 q_1 - \frac{ab}{\zeta_2} q_1^3$$

Active Brownian Particles in a Liquid





Hydrodynamical Interaction

local velocity induced by the surrounding particles in a laminar regime

$$v_{\mathrm{F}}(\boldsymbol{r}_i) = \sum_j \frac{R}{r_{ij}} \left[\boldsymbol{\delta} + \frac{\boldsymbol{r}_{ij} \otimes \boldsymbol{r}_{ij}}{r_{ij}^2} \right]$$

or

$$\boldsymbol{v}_{\mathrm{F}}(\boldsymbol{r}_i) = \sum_j \frac{R}{r_{ij}} \boldsymbol{v}_j + \sum_j \frac{R(\boldsymbol{r}_{ij} \cdot \boldsymbol{v}_j)}{r_{ij}^3} \boldsymbol{r}_{ij} \,; \quad r_{ij} > R$$

The Langevin equation for a single particle becomes:

$$\boxed{\dot{\boldsymbol{v}}_i = -\gamma(\boldsymbol{v}_i)\boldsymbol{v}_i + \kappa_{\mathrm{F}} \boldsymbol{v}_{\mathrm{F}}(\boldsymbol{r}_i) + \boldsymbol{F}(\boldsymbol{r}_i) + \sqrt{2D}\boldsymbol{\xi}_i(t).}$$

Erdmann and Ebeling, Fluctuation and Noise Letters **3**(2), L145-L154 (2003)

Fokker-Planck Equation

If the dynamics of a single particle is:

$$\dot{\boldsymbol{r}} = \boldsymbol{v}\,, \qquad \dot{\boldsymbol{v}} = -\gamma(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{F}(\boldsymbol{r}) + \sqrt{2D}\boldsymbol{\xi}(t)$$

The corresponding Fokker-Planck equation is for the PDF becomes:

$$\frac{\partial P}{\partial t} = -\boldsymbol{v}\,\frac{\partial P}{\partial \boldsymbol{r}} - \boldsymbol{F}(\boldsymbol{r})\,\frac{\partial P}{\partial \boldsymbol{v}} + \frac{\partial}{\partial \boldsymbol{v}}\left\{\gamma(\boldsymbol{v})\,\boldsymbol{v}\,P + D\,\frac{\partial P}{\partial \boldsymbol{v}}\right\}$$

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